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Magnetohydrodynamic Modeling of Self - Generated Magnetic Fields in Plasmas

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ABSTRACT

Space-time evolution of Self-Generated Magnetic Field (SMF) in plasmas is investigated analytically and numerically. The equation for the evolution of the magnetic field has been solved as an initial-value problem. For a general source that includes both thermal and radiative source and by accounting for retardation effects, expressions for the space-time development of the SMF have been obtained in a closed form, for both bounded plasmas and plasmas of infinite extent. Modelling of the self-generated magnetic field by an oscillating Gaussian source with a source strength equal to that of the thermoelectric source has been discussed. The peak values of the SMF for a typical laser - produced plasma are found to be $B \approx 8.5$ MG and $B \approx 4$ MG for the charge states Z= 10 and Z = 20, respectively.

Keywords: Plasma, Magnetic field generation, Magnetohydrodynamics.

1. INTRODUCTION

Strong magnetic fields have been observed in laser-produced plasmas and were first reported for a gas breakdown (Korobkin and Servo, 1966), and subsequently for a solid target (Askar'yan et al., 1967). Later, rather large fields, of the order of several kilogauss, were reported and explained in terms of thermal sources (Stamper et al., 1971; Raven et al., 1978). Extremely intense self-generated magnetic fields, of the order of hundreds of megagauss around the heated spot of the plasma, were observed (Raven et al., 1978; Stamper and Ripin, 1975; Bell et al., 1993; Boyd et al., 1996).

There are a variety of physical mechanisms and sources for the generation of large fields of the order of $\geq 10^6$ Gauss (Bell et al., 1993; Boyd et al., 1996; Stamper, 1991; Carxton and Haines, 1975; Haines, 1986; Sudan, 1993; Colombant and Winsor, 1977). During the process of laser-plasma interaction, magnetic field generation mechanisms such as noncolinear density (n) and temperature (T) gradients, resonance absorption and plasma magnetic instabilities (high Z-plasmas), can occur (Thomson et al., 1975). Diffusion and propagation of generated magnetic fields can strongly influence the electrons dynamics, which, in turn, are responsible for the field generation due to their high mobility. Plasma ions are very heavy and immobile; and therefore, are not responsible for the generation of such fields.

Deposition of magnetic field energy results from continuous changes in the electron gyration radius due to the feedback effect of continuous field generation, and electron thermal energy will, therefore, increase. Such magnetic fields can strongly affect energy transport leading to hot spots, fast electrons and fast ions (Carxton and Haines, 1978). Magnetic fields inhibit thermal

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conduction, and therefore, they influence temperature distributions and thereby parametric processes, energy absorption and laser beam propagation in plasmas (Forslund and Brackbill, 1982).

Forslund and Brackbill (1982) investigated magnetic fields that induced transport on laser-irradiated foils. Electrons heated by absorption of laser energy are shown to generate intense magnetic fields, which rapidly spread from the edge of the laser spot along the target surface. The field transports hot electrons convectively and confines a major fraction of the deposited laser energy in the corona. This model could explain many experimental observations of thermal transport inhibition as well as fast ion loss.

The effect of displacement current on the magnetic field propagation into the plasma in the form of a whistler wave, magnetic field penetration and electron heating in weakly nonuniform plasmas has been studied by Gomberoff and Fruchtman (Gomberoff and Fruchtman, 1992; Fruchtman and Gomberoff, 1992). In an azimuthally symmetric plasma, penetration was found to be caused by both convective skin effect and pressure gradient.

Saturation mechanisms for self - generated magnetic fields in nonuniform laser - matter irradiation were studied by Haines (1997). Tripathi and Liu (1992) studied the problem of self-generated magnetic field in an amplitude modulated laser filament in plasma and discussed the generation of a quasi-static field and its relevance to plasma accelerators and moving ionization fronts.

Self-generated magnetic fields play an essential role in the study of parametric instabilities (Bawa'aneh, 2003; Bawa'aneh, 2006), since magnetized plasma occurs in natural systems in space physics and the atmosphere as well as in the inertial confinement fusion in laboratory plasma. Recently, self-generated magnetic fields have been investigated thoroughly (Frederiksen et al., 2004; Ding et al., 2003; Brizard et al., 2000; Shvetz and Fisch, 2002) Frederiksen *et al.* (2004) have confirmed strong self - generated fields using a three-dimensional particle simulation on ion-electron counter streaming plasma. Experiments made by Ding *et al.* (2003) measured magnetic field fluctuations in the core of high-temperature reversed field pinch using a newly developed fast polarimetry system. Brizard *et al.* (2000) investigated the generation of magnetic fields from collective neutrino-plasma interaction, where the neutrino-plasma fluid equations are derived from a covariant relativistic variational principle. Shvets and Fisch (2002) have investigated the magnetic field generation through angular momentum exchange between circularly polarized radiation and charge particles by means of inverse Faraday effect. They found that dissipative mechanisms play a role in determining the efficiency of the field generation.

In this paper, we investigate the self-generated magnetic field in both bounded and unbounded plasmas. In Section 2, we present the equation governing the space-time development of the magnetic field. In Section 3, we solve the dynamic equation for bounded plasma as an initial-value problem to obtain an analytical expression in a closed form for the self-generated magnetic field. In Section 4, we obtain an expression for the self-generated magnetic field for an anisotropic, continuous plasma. In Section 5, we model the magnetic field generation in laser-produced plasma by oscillating profiles with a source strength equal to that of the thermoelectric source. In Section 6, we present the main conclusions.

2. MODEL EQUATIONS

The equation for the space - time evolution of the self-generated magnetic fields in plasmas is (Stamper et al., 1971; Stamper, 1991; Colombant and Winsor, 1977; Boyd and Cooke, 1988):

$$\frac{\partial \vec{B}}{\partial t} - \frac{\eta}{\mu_0} \vec{\nabla}^2 \vec{B} + \vec{\nabla} \times \left(\vec{v} \times \vec{B} \right) = \vec{S}(\vec{r}, t) \qquad (2.1)$$

$$\vec{S}(\vec{r}, t) = \vec{S}_t + \vec{S}_r = \frac{1}{e} \vec{\nabla} \times \left(\frac{1}{n_e} \vec{\nabla} \cdot \vec{P}_e \right) + \frac{1}{e} \vec{\nabla} \times \left(\frac{1}{n_e} \vec{\nabla} \cdot \vec{P}_r \right) \qquad (2.2)$$

where $\vec{S}(\vec{r}, t)$ is the net magnetic field source, $\vec{P}_e$ and $\vec{P}_r$ are the thermal and radiation pressure tensors, respectively, $\vec{v}$ is the fluid velocity, e is the electron charge and $\eta = \frac{m_e}{n_e e^2} v_{ei}$ is the specific resistivity of the plasma and v_{ei} is the electron - ion collision frequency. The second and third terms on the left-hand side of equation (2.1) are the diffusion and convective terms of the magnetic field.

The thermal source term $\vec{S}_t$ is nonlinearly modified by the presence of the radiation source term $\vec{S}_r$. Other source terms are present when kinetic, rather than fluid, description is used. In the regime where resistive effects are assumed to be dominant, the net source $\vec{S}(\vec{r}, t)$ on the right-hand side of equation (2.1) is balanced or saturated by the resistive effects that appear in the diffusion term. The regime where anomalous resistivity dose appear due to micro-instabilities and the regime where magnetic field generation is saturated by field convection, both are not included in equation (2.1).

It was pointed out by Boyd and Cooke (1988) that neither the Hall term nor the thermal force term can by themselves generate magnetic fields. Though, they can affect the morphology of these fields. Collisionless Hall forces associated with the ordinary and radiative currents describe the redistribution of the magnetic field (Stamper, 1991).

Assuming that the fluid drifts are parallel to the magnetic field, equation (2.1) becomes

$$\frac{\partial \vec{B}}{\partial t} - \frac{\eta}{\mu_0} \vec{\nabla}^2 \vec{B} = \vec{S}(\vec{r}, t) \qquad (2.3)$$

For plasmas in thermodynamic equilibrium, the thermal source reduces to $\vec{S}_t \approx \vec{\nabla} n_e \times \vec{\nabla} T_e$, which is the principal source for the magnetic field generation in the inhomogeneous anisotropic plasmas with nonaligned electron density and temperature gradients. The source $\vec{\nabla} n_e \times \vec{\nabla} T_e$ is capable of producing megagauss dc magnetic fields in large as well as in small scales (Boyd et al., 1996). For anisotropic plasmas with nonvanishing-off diagonal term of the electron pressure tensor, large scale magnetic fields are produced even for parallel $\vec{\nabla} n_e$ and $\vec{\nabla} T_e$.

The full equation for the space - time evolution of the self-generated magnetic field should include other possible terms such as magnetic field convection, magnetic curvature, magnetic pressure and the thermal force (Stamper et al., 1971; Stamper, 1991; Haines, 1986; Colombant and Winsor, 1977; Mora and Pella, 1981; Boyd and Cooke, 1988). It is well known that accounting for all these effects is a difficult task analytically as well as numerically.

3. INHOMOGENEOUS ANISOTROPIC BOUNDED PLASMAS

In this section, the space - time evolution of the self-generated magnetic field will be solved as an initial-value problem for bounded plasmas. For this purpose, we introduce the Fourier cosine transformation in three dimensions as follows:

$$\vec{B}(\vec{r}, t) = \sum_{nlm}^{\infty} \cos \frac{n\pi x}{L_x} \cos \frac{l\pi y}{L_y} \cos \frac{m\pi z}{L_z} \vec{B}(n, l, m, t) \qquad (3.1)$$

$$\vec{S}(\vec{r},t)=\sum_{nlm}^{\infty}\cos\frac{n\pi x}{L_x}\cos\frac{l\pi y}{L_y}\cos\frac{m\pi z}{L_z}\vec{S}(n,l,m,t) \quad (3.2)$$

$$\vec{B}(n,l,m,t)=\frac{8}{V}\int_V d^3r\cos\frac{n\pi x}{L_x}\cos\frac{l\pi y}{L_y}\cos\frac{m\pi z}{L_z}\vec{B}(\vec{r},t) \quad (3.3)$$

$$\vec{S}(n,l,m,t)=\frac{8}{V}\int_V d^3r\cos\frac{n\pi x}{L_x}\cos\frac{l\pi y}{L_y}\cos\frac{m\pi z}{L_z}\vec{S}(\vec{r},t), \quad (3.4)$$

where $V=L_xL_yL_z$ is a characteristic volume and L_x, L_y and L_z are characteristic scale lengths, equation (2.3) becomes

$$\left(\frac{\partial}{\partial t}+\frac{\eta}{\mu_0}\left[\left(\frac{n\pi}{L_x}\right)^2+\left(\frac{l\pi}{L_y}\right)^2+\left(\frac{m\pi}{L_z}\right)^2\right]\right)\mathbf{B}(n,l,m,t)=\vec{S}(n,l,m,t), \quad (3.5)$$

where the plasma resistivity η is taken to be uniform. Equation (3.5) will be solved for bounded plasmas as an initial-value problem for a given $\vec{\mathbf{B}}(n,l,m,t=0)$. Using Laplace transform of an arbitrary function $F(t)$,

$$L\{F(t)\}=f(p)=\int_0^{\infty}dt\ e^{-pt}F(t) \quad (3.6)$$

$$L\left\{\frac{\partial F(t)}{\partial t}\right\}=pf(p)-F(0), \quad (3.7)$$

equation (3.5) reduces to be:

$$\left[p+\frac{\eta}{\mu_0}\left(\left(\frac{n\pi}{L_x}\right)^2+\left(\frac{l\pi}{L_y}\right)^2+\left(\frac{m\pi}{L_z}\right)^2\right)\right]\vec{b}(n,l,m,p)-\vec{B}(n,l,m,t=0)=\vec{s}(n,l,m,p) \quad (3.8)$$

where $\vec{b}(n,l,m,p)$ and $\vec{s}(n,l,m,p)$ are the Laplace transforms of the Fourier transformed magnetic field $\vec{B}(n,l,m,t)$ source $\vec{S}(n,l,m,t)$, respectively. Solving equation (3.8) for $\vec{b}(n,l,m,p)$ gives:

$$\vec{b}(n,l,m,p)=\frac{\vec{B}(n,l,m,t=0)}{p+\frac{\eta}{\mu_0}\left[\left(\frac{n\pi}{L_x}\right)^2+\left(\frac{l\pi}{L_y}\right)^2+\left(\frac{m\pi}{L_z}\right)^2\right]}+\frac{\vec{s}(n,l,m,p)}{p+\frac{\eta}{\mu_0}\left[\left(\frac{n\pi}{Lx}\right)^2+\left(\frac{l\pi}{L_y}\right)^2+\left(\frac{m\pi}{L_z}\right)^2\right]} \quad (3.9)$$

Inversion of the Laplace transforms in equation (3.9) for $\vec{B}(n,l,m,t)$ using the convolution theorem

$$F(t)=L^{-1}\{f(p)\}=L^{-1}\{f_1(p)f_2(p)\}=\int_0^t d\tau\ \ F_1(t-\tau)F_2(\tau),$$

where $f_1(p)$ and $f_2(p)$ are the Laplace transforms of $F_1(t)$ and $F_2(t)$ respectively, gives:

$$\vec{B}(n,l,m,t)=\vec{B}(n,l,m,0)\ e^{-\frac{\eta}{\mu_0}\gamma^2 t}+\int_0^t d\tau\ e^{-\frac{\eta}{\mu_0}\gamma^2(t-\tau)}\vec{S}(n,l,m,\tau) \quad (3.10)$$

where γ^2 is given by

$$\gamma^2=\left[\left(\frac{n\pi}{L_x}\right)^2+\left(\frac{l\pi}{L_y}\right)^2+\left(\frac{m\pi}{L_z}\right)^2\right] \quad (3.11)$$

Equation (3.10) is causal in the sense that switching on a source at time $t'=\tau$ (action) will produce a magnetic field at a later time t (reaction). At time $t'=\tau=t$ the source reaches its maximum strength and the field at any time t is dependent on the source history $0\le\tau\le t$ (memory or retardation effect).

Substituting for $\vec{B}(n,l,m,t)$ from equation (3.10) into equation (3.1) gives:

$$\vec{B}(\vec{r},t)=\sum_{nlm}^{\infty}\vec{B}(n,l,m,0)\ e^{-\frac{\eta}{\mu_0}\gamma^2 t}\cos\frac{n\pi x}{L_x}\cos\frac{l\pi y}{L_y}\cos\frac{m\pi z}{L_z}+\sum_{nlm}^{\infty}\left[\int_0^t d\tau\ e^{-\frac{\eta}{\mu_0}\gamma^2(t-\tau)}\vec{S}(n,l,m,\tau)\right]\cos\frac{n\pi x}{L_x}\cos\frac{l\pi y}{L_y}\cos\frac{m\pi z}{L_z} \quad (3.12)$$

For a given initial magnetic field $\vec{B}(\vec{r},t=0)$ and source $\vec{S}(n,l,m,\tau)$, the magnetic field $\vec{B}(\vec{r},t>0)$ in bounded plasma is determined in a closed form according to equation (3.12). The functions $\vec{B}(n,l,m,0)$ and $\vec{S}(n,l,m,\tau)$ are to be evaluated using equation (3.3) and equation (3.4), respectively.

4. INHOMOGENEOUS ANISOTROPIC PLASMAS

We now solve equation (2.3) for the space-time evolution of the self-generated magnetic field as an initial-value problem for continuous plasmas. For this pupose, we introduce the Fourier transformation in space only as follows,

$$\vec{B}(\vec{r},t)=\frac{1}{(2\pi)^3}\int d^3k \;\; e^{i\vec{k}.\vec{r}}\,\vec{B}(\vec{k},t) \tag{4.1}$$

$$\vec{S}(\vec{r},t)=\frac{1}{(2\pi)^3}\int d^3k \;\; e^{i\vec{k}.\vec{r}}\,\vec{S}(\vec{k},t) \tag{4.2}$$

$$\vec{B}(\vec{k},t)=\int d^3r \;\; e^{-i\vec{k}.\vec{r}}\,\vec{B}(\vec{r},t) \tag{4.3}$$

$$\vec{S}(\vec{k},t)=\int d^3r \;\; e^{-i\vec{k}.\vec{r}}\,\vec{S}(\vec{r},t) \tag{4.4}$$

Fourier transforming equation (2.3) in space only gives:

$$\frac{\partial\vec{B}(\vec{k},t)}{\partial t}+\frac{\eta}{\mu_0}k^2\vec{B}(\vec{k},t)=\vec{S}(\vec{k},t) \tag{4.5}$$

Equation (4.5) will be solved as an initial-value problem for a given $\vec{B}(\vec{k},t=0)$. Using Laplace transform, eqn. (4.5) reduces to be:

$$\left[p+\frac{\eta}{\mu_0}k^2\right]\;\vec{b}(\vec{k},p)-\vec{B}(\vec{k},t=0)=\vec{s}(\vec{k},p), \tag{4.6}$$

where $\vec{b}(\vec{k},p)$ and $\vec{s}(\vec{k},p)$ are the Laplace transforms of $\vec{B}(\vec{k},t)$ and $\vec{S}(\vec{k},t)$, respectively. Solving equation (4.6) for $\vec{b}(\vec{k},p)$ gives:

$$\vec{b}(\vec{k},p)=\frac{\vec{B}(\vec{k},t=0)}{p+\frac{\eta}{\mu_0}k^2}+\frac{\vec{s}(\vec{k},p)}{p+\frac{\eta}{\mu_0}k^2} \tag{4.7}$$

Inversion of the Laplace transforms in equation (4.7) for $\vec{B}(\vec{k},t)$ using the convolution theorem gives:

$$\vec{B}(\vec{k},t)=\vec{B}(\vec{k},0)\;e^{-\frac{\eta}{\mu_0}k^2t}+\int_0^t d\tau\;e^{-\frac{\eta}{\mu_0}k^2(t-\tau)}\vec{S}(\vec{k},\tau) \tag{4.8}$$

Equation (4.8) is causal in the sense that switching a source on at time $t'=\tau$ (action) will produce a magnetic field at a later time t (reaction). At time $t'=\tau=t$, the source reaches its maximum strength and the field at any time t is dependent on the source history $0\le\tau\le t$ (memory or retardation effect).

Substituting for $\vec{B}(\vec{k},t)$ from equation (4.8) into equation (4.1) gives:

$$\vec{B}(\vec{r},t)=\frac{1}{(2\pi)^3}\int d^3k\;\;\vec{B}(\vec{k},0)\,e^{-\frac{\eta}{\mu_0}k^2t}\,e^{i\vec{k}.\vec{r}}+\frac{1}{(2\pi)^3}\int d^3k\left[\int_0^t d\tau\;\;e^{-\frac{\eta}{\mu_0}k^2(t-\tau)}\vec{S}(\vec{k},\tau)\right]e^{i\vec{k}.\vec{r}} \tag{4.9}$$

The source $\vec{S}(\vec{k},t)$ is assumed to be switched on adiabatically at time $\tau=0$ and it will reach its maximum strength at $\tau=t$. The process of field generation is therefore nonlocal in time. The first term on the right-hand side of equation (4.9) represents the transient field component that dies out as t becomes large. The second term on the right-hand side of equation (4.9) is the steady-state component of the generated magnetic field.

5. OSCILLATING SOURCE OF MAGNETIC FIELD

In this section, eqn. (4.9) will be used for modeling the self-generated magnetic field for the case of an oscillating source. Assuming no initial field, $\vec{B}_0(\vec{r},t=0)=0$, and that a source term of the form,

$$\vec{S}(\vec{r},t)=\vec{S}_0\,f(\omega_0t)\;\;e^{-ar^2} \tag{5.1}$$

exists on the plasma boundary, that is eventually advected inside the plasma (Boyd et al., 1996), where S_0 is the source amplitude with the dimension of the magnetic field per unit time, f is the time part of the source, a is constant and ω_0 is a characteristic oscillation frequency of the magnetic field source. With these assumptions, equation (4.9) becomes:

$$B(\vec{r},t) = S_0 \frac{1}{(2\pi)^3}\left(\frac{\pi}{a}\right)^{\frac{3}{2}} \int_0^t d\tau \quad f(\omega_0\tau) \int d^3k \left[e^{-\frac{\eta}{\mu 0}k^2(t-\tau)} e^{i\vec{k}.\vec{r}} e^{-\frac{k^2}{4a}} \right] = \frac{S_0}{\pi}\int_0^t d\tau \frac{f(\omega_0\tau)}{\left[1+\frac{4\eta a(t-\tau)}{\mu_0}\right]^{\frac{3}{2}}} \exp - \left[\frac{ar^2}{1+\frac{4\eta a(t-\tau)}{\mu_0}} \right] \quad (5.2)$$

For a given typical magnetic field source S_0, eqn. (5.2) determines the generated magnetic field at $\vec{r}$ and t. After specifying the time dependence of the source term $f(\omega_0 t)$, equation (5.2) determines the self-generated magnetic field in space and time. In the rest of the section, two time functions, that are widely used in literature, are considered; $f(\omega_0 t) = \sin\omega_0 t$ and $f(\omega_0 t) = \sin^2\omega_0 t$. The reason for using these two profiles comes as a numerical necessity where the source is switched at some time t=0 to reach it's maximum value adiabatically at a later time. In the following numerical analysis, we will assume that the source amplitude S_0 is in of the order of magnitude of the amplitude of the thermal source $\frac{k_B T_e}{eL_n L_T}$, where L_n and L_T are the density and temperature gradient scale lengths, respectively (Stamper, 1991; Haines, 1986; Colombant and Winsor, 1977).

For laser-produced plasmas of the typical density $n_e \approx 10^{26} m^{-3}$, electron temperature $T_e \approx 200$ eV and electron Debye length $\lambda_D \approx 10^{-9} m$, we plot the self-generated magnetic field as a function of time for $Z = 10$ and $Z = 20$ (Figures 1 and 2). For the same plasma parameters the self-generated magnetic field has been plotted as a function of the radial position *r* (Fig. 3). For charge state of $Z = 10$ the Coulomb logarithm and plasma resistivity are found self consistently in the code to be $\log\Lambda = 12.45$ and $\eta = 4.45\times10^{-4}\,\Omega m$. The Coulomb logarithm and plasma resistivity for $Z = 20$ are found to be $\log\Lambda = 12.15$ and $\eta = 8.85\times10^{-4}\,\Omega m$.

Figures (1 and 2), which represent the time evolution of the magnetic field, show that the field saturates when increasing time. For $f(\omega_0 t) = \sin\omega_0 t$, the saturation field values are $B_{sat} \approx 8.5$, 4 MG for $Z = 10$ and $Z = 20$, respectively. For $f(\omega_0 t) = \sin^2\omega_0 t$ the saturation field values are $B_{sat} \approx 9.25$, 3.6 MG for $Z = 10$ and $Z = 20$, respectively.

Fig. (3), which represents the magnetic field as a function of the radial position for $f(\omega_0 t) = \sin\omega_0 t$, shows decay in the magnetic field from a peak value at $r = 0$ to zero at the high values of *r*. This is an expected result where the field takes a maximum at $r = 0$ and drops to zero at large *r*. The plasma dimensions used here are long enough to justify considering homogeneous plasma. The time $T_0 = 2\pi\omega_0{}^{-1}$ is taken to be $6\times10^{-9} s$, α is equal to $35T_d$ and $70T_d$ for $Z = 10$ and $Z = 20$, respectively, where T_d is a characteristic time defined by

$$T_d = \frac{\mu_0 R_0{}^2}{4\eta}, \; R_0 = \sqrt{a^{-1}} = 5\times10^{-4} m.$$

6. CONCLUSION

The equation that describes the space-time development of the self-generated magnetic fields in plasmas has been presented, and the conditions for its validity have been discussed. As principal sources for the magnetic field generation, two general sources are considered; a radiative source and a thermal source on the boundary of a laser-produced plasma generated by the nonaligned density and temperature gradients made by a high power laser hitting the plasma boundary.

The usual thermal source term $\vec{S}_t$ is nonlinearly modified by the presence of the radiation source term $\vec{S}_r$. Other source terms are present when a kinetic, rather than a fluid, description is used (Haines, 1986). Collisionless Hall forces associated with ordinary and radiative currents describe the redistribution of the magnetic field. As pointed out in literature, neither the Hall term nor the thermal force term by themselves generate magnetic fields. Though, they can affect the morphology of these fields.

The basic equation for the magnetic field evolution is solved as an initial-value problem by adopting Fourier transformation in space and Laplace transformation in time. For a given initial magnetic field $\vec{B}(\vec{r}, t_0)$, thermal source $(\vec{S}_t \approx \vec{\nabla}.\vec{P}_t)$ and radiative source $(\vec{S}_t \approx \vec{\nabla}.\vec{P}_r)$, the magnetic field $\vec{B}(\vec{r}, t > t_0)$ can be determined in a closed form. We calculated the magnetic field $\vec{B}(\vec{r}, t)$ for both bounded and unbounded plasmas [see equations (3.15) and (4.24)].

We have shown that for bounded plasmas and for plasmas as a continuous medium, the magnetic field source is assumed to be switched on adiabatically at time $\tau = 0$ and that it will reach its maximum strength at $\tau = t$.

To model the magnetic field generation, two examples of a source term of the form of oscillating Gaussian profiles, namely; $f(\omega_0 t) = \sin \omega_0 t$ and $f(\omega_0 t) = \sin^2 \omega_0 t$, are considered. The source strength S_0 is taken to be equal to that of the thermoelectric source $S_0 = \frac{k_B T_e}{e L_n L_T}$. Figures (1 and 2) represent the time evolution of the magnetic field. They show field saturation with increasing the time. The saturation field values are $B_{sat} \approx 8.5,\ 4\,\text{MG}$ in case of $f(\omega_0 t) = \sin \omega_0 t$ and $B_{sat} \approx 9.25,\ 3.6\,\text{MG}$ in case of $f(\omega_0 t) = \sin^2 \omega_0 t$ for $Z = 10$ and $Z = 20$, respectively.

Both time functions $f(\omega_0 t) = \sin \omega_0 t$ and $f(\omega_0 t) = \sin^2 \omega_0 t$ show the same behavior for the magnetic field in the radial direction. As an example, Fig.3 shows the decay of the magnetic field for time function $f(\omega_0 t) = \sin \omega_0 t$. Morphology of the resulting fields is in agreement with results obtained in (Boyd et al., 1996) generated thermoelectrically.

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النمذجة الهيدروديناميكية للمجالات المغناطيسية المتولدة في البلازما

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ملخص

قمنا بدراسة تغير المجال المغناطيسي المتكون في البلازما من وجهة النظر النظرية والحاسوبية. وتم اشتقاق وحل المعادلات التي تحكم سلوك المجال المغناطيسي بالنسبة للبلازما المحددة والبلازما الممتدة، آخذين بعين الاعتبار العوامل المعروفة للبلازما، وهيدروديناميكية المجالات المغناطيسية، والحصول على قيم قصوى للمجال المغناطيسي مقدارها 8.5 و4 ميغاغاوس بالنسبة للعددين الذريين 10 و20 على التوالي.

الكلمات الدالة: البلازما، المجال المغناطيسي، وهيدروديناميكية المجالات المغناطيسية.

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