Dynamic Model of a Single–phase Induction Motor Using an Electronic Triac-starter

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ABSTRACT

In this paper a theoretical dynamic model of a single-phase induction motor employing an innovative electronic triac-starter is derived with a stationary d-q reference frame, using Matlab/simulink software package program. The obtained theoretical results were experimentally verified using a specially designed testing station. This new starting method is used to replace the conventional capacitor starter in series with the auxiliary winding of a single-phase induction motor, employing the optimal firing angle which facilitates starting-up without capacitor. It was concluded that the theoretical and experimental results are in full agreement, and confirmed that this new starting scheme can substitute the conventional capacitor without affecting the performance of the studied single-phase induction motor.

Keywords: Single-phase induction motor, Electronic circuit, Dynamic model, Triac-starter.

1. INTRODUCTION

Single-phase Induction Motors (SPIM), are usually low power machines and widely used in industry and residential appliances. This type of motors cannot be started directly from the mains because of its poor required starting torque. Thus, the single–phase induction motor is normally operated with an auxiliary windings having inductive or capacitive characteristic. The latter is widely used by manufactures due to high performance and low costs. In the market, SPIM can be found either with single or double capacitors. In the single capacitor system, the capacitor is taken out together with the auxiliary winding at about 75% of the synchronous speed of SPIM. While in the double capacitor system, one capacitor is driven out at 75% of the synchronous speed and the other one remains continuously in the operation. By this way the starting and running torque of the SPIM is improved (Ozdemir et al., 1998). Capacitance variation can be eliminated by using adjustable AC capacitor (Muljadi et al., 1993; Lettenmaier et al., 1991; Faiz, 2004), which enables to select the optimum capacitance value for any desired rotor speed. A recent paper presented and discussed the new electronic circuit that replaces the conventional capacitor, without affecting the general performance of the SPIM (Jaber, 2006). It was proved that employing a triac-switch, with a firing angle of about 90°, will replace the capacitor and start the motor easily and smoothly by providing the required starting torque.

In the literature few studies that used the stationary d–q reference frame model for SPIM were traced. It has been used by some researchers for an integral–cycle controlled SPIM (Longya, 1992; Al-Turki and Al-Umari, 2000), by utilizing this model it is revealed that two types of capacitor connection to the machine windings are equivalent and the integral-cycle controlled two distinct modes. While other investigators used the dynamic model of the SPIM with the switching capacitor (Sunter and Ozdemir, 2002). Where in this model the drive is used to obtain the maximum torque by employing optimum capacitor values in the SPIM with switching capacitor.

In a previous paper, mathematical static-analysis and different sets of experiments aiming to test a novel method to substitute for the traditional capacitor-start single-phase induction motor, were presented and discussed (Jaber, 2006). While the current paper presents a dynamic-model of a single-phase induction motor. This investigation was conducted into two steps. The first was

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concerned with developing the dynamic model of the studied SPIM with a triac-starter, using Matlab/simulink package program. While the second one represents the experimental work to confirm the obtained results from the dynamic model. Those who are interested in more details about the Matlab package, it is recommended to consult the Matlab for Microsoft Windows, which could be found elsewhere (Matlab for Microsoft Windows, 1999).

Mathematical Model

The single-phase induction motor circuit using a triac-starter in series with auxiliary winding is shown in Fig.(1). It is clear from the principle circuit that the triac-starter consists of anti-parallel thyristors with firing currents, i.e. \( I_g \).

\[
\begin{align*}
\frac{d\Psi_{sd}}{dt} &= V_{sd} - r_{sd}i_{sd} \quad \ldots (1) \\
\frac{d\Psi_{sq}}{dt} &= V_{sq} - r_{sq}i_{sq} \quad \ldots (2) \\
\frac{d\Psi_{rd}}{dt} &= -r_{rd}i_{rd} - \omega_r \Psi_{rq} \quad \ldots (3) \\
\end{align*}
\]

\[
\begin{align*}
\frac{d\Psi_{rq}}{dt} &= -r_{rq}i_{rq} + \omega_r \Psi_{rd} \quad \ldots (4) \\
i_{sd} &= \frac{L_z \Psi_{sd}}{A} - \frac{L_m}{A} \Psi_{rd} \quad \ldots (5) \\
i_{rd} &= \frac{-L_m}{A} \Psi_{rd} + \frac{L_{sd}}{A} \Psi_{rd} \quad \ldots (6) \\
i_{sq} &= \frac{L_z}{B} \Psi_{sq} - K^2 \frac{L_m}{B} \Psi_{rq} \quad \ldots (7) \\
i_{rq} &= -\frac{K^2 L_m}{B} \Psi_{sq} + \frac{L_{sq}}{B} \Psi_{rq} \quad \ldots (8)
\end{align*}
\]

Where; \( V_{sq} \) and \( V_{sd} \) are the q and d axis stator voltages, \( V_{\text{switch}} \) is the voltage drop across the triac, \( r_{sq} \) and \( r_{sd} \) are the q and d axis stator resistances, \( r_r \) the rotor resistance, \( L_{sq} \) and \( L_{sd} \) are the q and d axis stator leakage inductances, \( L_r \) the rotor leakage inductance, \( i_{sq} \) and \( i_{sd} \) are the q and d axis stator currents, \( (d/dt) \) is the deferential operator, \( \psi_{sq}, \psi_{sd}, \psi_{rq} \) and \( \psi_{rd} \) are the q and d axis stator flux linkage and rotor flux linkage resolved to r-phase quantities, \( L_m \) is the q and d axis mutual inductances, \( \omega_r \) is the rotor angular speed. All values of these parameters, used in this study, are shown in Table (1). The K, A and B coefficients can be expressed as follow:
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\[
\begin{align*}
K &= \frac{W_{sd} K_d}{W_{sq} K_q} \quad \text{... (9)} \\
A &= L_{sd} \ell_r - L_{em} \quad \text{... (10)} \\
B &= L_{sq} \ell_r - K^2 L_{em} \quad \text{... (11)} \\
L_{sd} &= \ell_{sd} + L_{m} \quad \text{... (12)} \\
L_{sq} &= \ell_{sq} + K^2 L_{m} \quad \text{... (13)} \\
L_{sr} &= \ell_r + L_{m} \quad \text{... (14)}
\end{align*}
\]

Where; \( W_{sq} \) and \( W_{sd} \) are the q and d axis number of turns of stator windings, \( K_q \) and \( K_d \) winding coefficients, \( \ell_{sq} \) and \( \ell_{sd} \) are the q and d axis self stator inductances, \( \ell_r \) is the self rotor inductance. The instantaneous, electromagnetic torque is calculated as:

\[
T_e = PL_{m} \left[ K_q i_{sq} i_{dr} - i_{sd} i_{q} \right] \quad \text{... (15)}
\]

Where: \( p \) is the number of pole pairs.

The electromechanical equation of the SPIM is:

\[
\frac{d\omega_r}{dt} = \frac{P}{J_m} \left[ T_e - T_L \right] \quad \text{... (16)}
\]

Where \( J_m \) is the inertia constant of the motor and load and \( T_L \) is the external load torque. In these equations the d-axis corresponds to the main winding and the q-axis represents the auxiliary winding. Therefore, \( V_{sd} \) voltage will be equal to the source voltage, \( V_{source} \). The auxiliary winding voltage is expressed as:

\[
V_{sq} = V_{source} - V_{\text{switch}} \quad \text{... (17)}
\]

Where \( V_{source} \) is the input voltage of the SPIM and \( V_{\text{switch}} \) is the voltage drop of the triac circuit connected to the auxiliary winding in series.

**Dynamic Modeling**

Matlab/Simulink software package program, which consists of standard toolboxes that enable to simulate actual engineering systems, based on differential equations (1-16) and equation (17) has been used to obtain the dynamic model of the SPIM in d-q axis of the studied system in this paper – see Fig (3). As can be seen from Fig (3), the triac-starter was modeled together with the SPIM, which makes it easier to compute motor's characteristics, i.e. input and output. The inputs include source and output voltage of auxiliary winding with triac-starter, while outputs comprise the d-q axis stator current, motor's torque and speed. The employed parameters, i.e. the d-q axis stator and rotor resistors as well as inductances, mutual inductances, number of poles and turn ratio, are shown inside blocks of Fig (3).

The detailed electronic triac-starter connected with the auxiliary winding of the SPIM is shown in Fig (4), and the firing angle has been selected to acquire the maximum starting torque as concluded from a previous study (Jaber, 2006). The pulse generator provides pulses required for firing thyristors, and the delay angle was selected from the pulse cycle and delay time of the pulse generator.

![Figure 3. Dynamic model of the SPIM drive system with electronic triac starter in simulink.](image-url)
2. SIMULATION RESULTS

The dynamic performance of the SPIM with a capacitor starter ($c = 30 \mu F$) and electronic triac-starter under no load are shown in Figs (5-8). Fig (5) illustrates the speed characteristic for the SPIM with capacitor (30 μF) in acceleration mode, while the speed characteristic for the SPIM with electronic triac-starter in acceleration mode is shown in Fig (6). As can be seen these figures, the SPIM with electronic triac-starter has a smooth transient characteristic than that of the capacitor starter. However, the conventional capacitor is faster than the SPIM with triac-starter: only about 0.15 second was required to reach steady state compared with 0.5 second in the case of the SPIM with triac-starter. Such a long time is needed to the presence of thyristors in the electronic triac-switch which delays the voltage of auxiliary winding. Similar findings observed for currents of the main winding as shown in Figs. (7 and 8).

The dynamic performance of the SPIM, in acceleration, under load is shown in Figs (9-12). It is clear that the acceleration time of the SPIM with capacitor starter is longer than that of the SPIM with electronic triac-starter, but with pulsating characteristics as shown in Fig. (9). In general, it is worth noting that the obtained results when the SPIM was loaded are completely different than those taken under no loads: triac-switch reached steady state operation in a shorter time of about 0.15-0.20 second – see Fig. (10), and time and current values of main winding were similar as can be seen in Figs (11 and 12).

3. EXPERIMENTAL SETUP AND RESULT

A single-phase motor, MS400-type, was selected randomly from the electric-machines laboratory in the Faculty of Engineering Technology, Al-Balqa’ Applied University, with main parameters shown in Table (1).

The control unit (for 1000 W pendulum machine, Model 67 10 610/67 10 611, made by ELWE Training Systems, Germany) has been designed to operate the pendulum machine driven by a 1000 W motor only (Elwe Training Systems for Basic and Future Oriented Education in Natural Science and Engineering, 1996) – see Fig. (13). This combination allows us to design and conduct various experiments aiming to test electrical machines in all four quadrants. The sequence for the operation of the automatic function as follows:

1. Adjusting the start speed.
2. Adjusting the stop speed.
3. Setting the function selector on "start".
4. Switching on the motor of the tested machine (pressing push–button start).
5. Recording the speed and torque and all parameters of SPIM.

To start with, it is important to note that the obtained experimental results are in full agreement with simulation result’s. In this section, similar approach was followed as in the simulation part in order to confirm and verify results. Fig (14) shows the acceleration speed of the SPIM for two cases, i.e with capacitor and the triac-switch. It can be seen that the use of electronic triac-switch makes the transient state smoother, but slower than the transient state of the SPIM with conventional capacitor, noting that in both cases no
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load was applied. This can be attributed to the fact that the start time required by the electronic triac-starter is slightly greater than that when starting capacitor was used. It is important to note that there is a full agreement between the predicted theoretical and obtained experimental results. Fig. (15) shows a comparison of starting performance using the starting capacitor and the electronic triac-starter under a torque of 4.8 Nm. It is obvious that the use of the electronic triac-switch produces enough accelerating torque as compared with conventional capacitor. Thus, the later can be replaced by the triac-switch with no fears from starting difficulties. Figs. (16 and 17) show the current wave of acceleration of SPIM under zero load torque and under constant torque of about 4.8 Nm, respectively. This experimental investigation, is one of an on-going practical research in the Department of Mechatronics at Al-Balqa' Applied University, confirms that the suggested mathematical model reflects the dynamic performance of the SPIM with triac-starter. Such a new scheme is deemed to exhibit lower price of SPIM in the market, thus consumers would enjoy using cheaper and reliable appliances.

Figure 5. Speed characteristic for the SPIM with starting capacitor \((c=30 \mu F)\) under zero load.

Figure 6. Speed characteristic for the SPIM with electronic triac starter under zero load.
Figure 7. Main winding current of SPIM with starting capacitor ($c=30 \, \mu F$) under zero load.

Figure 8. Main winding current of SPIM with electronic triac starter under zero load.

Figure 9. Simulation result of speed for the SPIM with starting capacitor and with 4.8 Nm load.
Figure 10. Simulation result of speed for the SPIM with electronic triac starter and with 4.8 Nm load.

Figure 11. Simulation result of main winding current with starting capacitor (30 µf) and with 4.8 Nm load.

Figure 12. Simulation result of main winding current with electronic starter and with 4.8 Nm load.
Table 1. Main parameters of the studied motor.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Designated value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Input Power (W)</td>
<td>1000</td>
</tr>
<tr>
<td>Nominal Power (W)</td>
<td>750</td>
</tr>
<tr>
<td>Speed (rpm)</td>
<td>1430</td>
</tr>
<tr>
<td>No. of poles</td>
<td>4</td>
</tr>
<tr>
<td>Rated voltage (V)</td>
<td>220</td>
</tr>
<tr>
<td>Auxiliary winding resistance (Ω)</td>
<td>11.5</td>
</tr>
<tr>
<td>Main winding resistance (Ω)</td>
<td>3.5</td>
</tr>
<tr>
<td>Secondary resistance (Ω)</td>
<td>1.25</td>
</tr>
<tr>
<td>Leakage reactance of auxiliary winding (Ω)</td>
<td>1.327</td>
</tr>
<tr>
<td>Leakage reactance of main winding (Ω)</td>
<td>0.26</td>
</tr>
<tr>
<td>Magnetic reactance (Ω)</td>
<td>65</td>
</tr>
<tr>
<td>Leakage reactance of secondary winding (Ω)</td>
<td>4.2</td>
</tr>
<tr>
<td>Moment of inertia (NmS²)</td>
<td>0.00348</td>
</tr>
<tr>
<td>Capacitance (μF)</td>
<td>30</td>
</tr>
<tr>
<td>Rated torque (Nm)</td>
<td>5.1</td>
</tr>
<tr>
<td>Max torque (Nm)</td>
<td>6.9</td>
</tr>
<tr>
<td>Auxiliary/Main turns ratio</td>
<td>2</td>
</tr>
<tr>
<td>Calculated motor parameters</td>
<td></td>
</tr>
<tr>
<td>( r_{sd \text{ aux}} ): 11.5 (Ω)</td>
<td></td>
</tr>
<tr>
<td>( r_{sd \text{ main}} ): 3.5 (Ω)</td>
<td></td>
</tr>
<tr>
<td>( \ell_{sd} ): 0.0042 (H)</td>
<td></td>
</tr>
<tr>
<td>( \ell_{sq \text{ main}} ): 0.01356 (H)</td>
<td></td>
</tr>
<tr>
<td>( r_r ): 1.25 (Ω)</td>
<td></td>
</tr>
<tr>
<td>( \ell_r ): 0.013564 (H)</td>
<td></td>
</tr>
<tr>
<td>( L_m ): 0.2 (H)</td>
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</tr>
<tr>
<td>( L_{sd} ): 0.2042 (H)</td>
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<tr>
<td>( L_{sq} ): 0.0635 (H)</td>
<td></td>
</tr>
<tr>
<td>( L_{rr} ): 0.213536 (H)</td>
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</tr>
</tbody>
</table>

4. CONCLUSION

A dynamic simulation of the SPIM with electronic triac-starter has been presented in this theoretical and experimental investigation. It has been shown that it is possible to obtain the maximum torque by using electronic triac-starter as compared to conventional capacitor without any difficulty. In this study, it was concluded that in the case of SPIM with electronic triac-starter the transient state is smoother, but slower, than the transient state of the SPIM with conventional capacitor at zero-load. Such finding could be attributed to the fact that the start time required by the electronic triac-starter, is slightly greater than that when starting capacitor was used. But it was found that the opposite occurred when the SPIM was loaded. Again in this study, the experimental part confirms the calculated theoretical characteristics of the studied SPIM system. It is believed that such a new starting system would improve the performance of these motors, hence, consumers and industry may benefit from the obtained results and advantages.
Figure 13. The experimental rig and control unit.

Figure 14: Acceleration of speed of SPIM under Zero load torque (1 cm = 100 ms); (1 cm = 32 rad/s)

(a: using capacitor starter (30 µf))

(b: with electronic starter).
(b: using electronic triac starter).

Figure 15. Acceleration of speed of SPIM under 4.8 Nm load torque (1 cm = 200ms); (1 cm 32 rad/s)

(a: using capacitance = 30 µF)
(b: using electronic triac stator)

Figure 16. The current wave of acceleration of SPIM under zero load torque (1 cm = 200ms) (1 cm = 25 A).

(a: using starting capacitor = 30 µF)
(b: using electronic triac starter)

Figure 17. Current waves of acceleration of SPIM under 4.8 Nm load torque (1 Cm = 200 ms) (1 cm = 25 A).
REFERENCES


MatLab for Microsoft Windows (the math works Inc., 1999).


Transactions on Industry Applications, 27 (1).

المؤثرات الديناميكية لمحرك حثي أحادي التيار مع بادئ حركة إلكتروني (تيراك)

قائمة جاير

ملخص

تم في هذه الورقة البحثية استخدام النموذج الديناميكى لمحرك حثي أحادي التيار، ويستخدمن في النموذج بادئ حركة لإلكتروني من نوع تيراك؛ وذلك باستخدام هيئة مرجعية ساكنة (d-q)، وباستخدام برامجيات Matlab/simulink، لتتأكد من النتائج المحركة في مقارنة النتائج النظرية بالنتائج العملية، وكذلك مقارنتها مع نتائج الطريقة التقليدية التي تستخدمن في نطاق المحرك، وتتبيين أن نتائج تطابقًا بين النتائج العملية للطريقة الجديدة بوجود تيراك، وتتباين النتائج الطريقة التقليدية في إثالة المحرك باستخدام المواضع، وفي الطريقة الجديدة استبدل المواضع بدائي إلكتروني دون أي ثور سليم في الأداء الديناميكى لمحرك حثي أحادي التيار.

الكلمات الدالة: محرك حثي أحادي التيار، الدائرة الكهربائية، النموذج الديناميكى، بادئ حركة (تيراك).