Modeling Partially Saturated Soil

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ABSTRACT

Many geotechnical problems involve the presence of partially saturated soil zones, the most common case being that of capillary zones above groundwater table. These zones are usually ignored in practice, and the soil is assumed to be either fully saturated or completely dry. This simplification is in many cases conservative since it does not take into account the increase in shear strength due to partial saturation.

The object of this paper is to reach a reasonable understanding and explanation of the fundamental behavior of partially saturated soil such as suction and shear strength. This can be achieved by analyzing the micromechanical forces that affect the soil particles from a theoretical point of view.

Keywords: Shear Strength, Unsaturated Soil, Suction and Micromechanical Force.

1. INTRODUCTION

The behavior of unsaturated soils has been characterized in terms of the suction in a sample, with good results for high degree of saturation where the pore water is continuous. Matric suction is attributed to capillary action in the soil structure, and it varies with changes in the moisture content of the soil. Pore fluid osmotic suction is related to the dissolved salt content in pore water (pore water salinity) and increases with pore water salinity. Constitutive models have been developed (e.g. Toll and Ong 2003) which can be used to describe the generalized behavior or the behavior of specific soil when sufficient testing has been carried out. However, at a low saturation the air phase is continuous, and the suctions become high and difficult to measure. The degree of saturation, on the other hand, is simple to measure.

At higher degrees of saturation, questions such as whether the soil is wet or dry, and the time taken for the aggregation of air bubbles, become at least as important as the suction or overall degree of saturation in determining the soil’s behavior. The air in such a soil will tend to aggregate, so that the soil becomes separated into regions of full saturation and low saturation. Samples could be prepared at a high degree of saturation, in which the air is at first uniformly distributed, but this will be a temporary state, and the ease with which air can move through the pore spaces determines how long this temporary state will last. In fine grained soils, this time will be long enough to allow investigation.

The shape of the meniscus and the pore geometry are important in understanding the behavior of unsaturated soil (Keen, 1923). Maaitah (2002) proposed a model by assuming that the meniscus shape is parabolic which made a great progress in understanding how the inter-particle acts in unsaturated media. Also, in (2004) he discussed the same problem by assuming that the shape of the meniscus is ellipse. In this paper, the section of the meniscus is assumed to be not exactly circular (i.e. the surface of the meniscus lies on a tours), it is sufficiently close to the circular. However, this assumption is more realistic, easier, and gives better result in comparison with experimental findings. Thorough investigation of the behavior is attempted here in order to understand the effect of packing, meniscus length, and contact angle. The particle packing factor concept is presented to show the effect of different sizes which allow the shear strength to be predicted more accurately for real soil samples. However, the object of the present work was to

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investigate the behavior of unsaturated soil at a low saturation in which approximately homogeneous conditions are reasonably stable. The more significant charter of this model is the ability for development that is going to be used in predicting the shear strength of unsaturated soil with clay size.

2. BEHAVIOR OF UNSATURATED SOIL

Low annual rainfall often leads to the development of desiccation cracks exceeding the depth of shallow footings. Also, the desiccation can be due to evaporation (i.e. a process which requires molecules to overcome surface tension). This problem is also responsible for the ground heaving due to an increase in moisture content. Whenever clay shrinkage or heave is suspected to cause damage to a property, four factors need to be addressed: water extracts from soil by trees, water supply to the soil from the ground surface, water flow within the soil and shrinkage or swelling of the clay.

The shear strength of dry or fully saturated cohesionless soils is derived from friction between soil particles at the points of contact; this shall be affected by tendency of the soil to expand against or contract with confining pressures as shearing takes place, until a critical state has been reached. Once the soil has reached a critical state, the shear strength is linearly related to the normal stress, and is usually expressed as:

$$\tau_{\max} = \sigma' \tan \phi'$$  \hspace{1cm} \text{(1)}

In which the effective stress, $\sigma'$, expresses the observation that if the soil is fully saturated, it is the difference between the total stress and the water pressure which leads to the shear strength. That is, the water pressure carries a proportion of the externally applied normal stress as if it were acting on the full area of any cross-section. This can easily be demonstrated to be a very good approximation for larger particles, for which the effective stress is the inter-granular stress.

The thermodynamic definition of wetting is based on the concept of surface energy or surface tension. Surface tension results from an imbalance of molecular forces in a liquid. At the surface of the liquid, the liquid molecules are attracted to each other and exert a net force pulling themselves together. High values of the surface tension means the molecules tend to interact strongly. Lower values mean the molecules do not interact as strongly.

Water has a very high value of surface tension because it has a high degree of hydrogen bonding (Mitchell, 1993).

In unsaturated soil, the frictional behavior of the soil-to-soil contacts is the same as in dry or saturated soils, and it would be possible to predict the shear strength in the same way if it were not for the following factors:

- The forces arise from the surface tension. The latter is an effect within the surface layer of a liquid that causes the layer to behave as an elastic sheet and causes capillary action. Surface tension is caused by the attraction between the molecules of the liquid, due to various intermolecular forces. The latter are electromagnetic forces which act between molecules or between widely separated regions of a macromolecule. Listed in order of decreasing strength, these forces are: (Ionic interactions, Hydrogen bond, dipole-dipole interactions and London Dispersion Forces (Mitchell, 1993)). The air pressure and the water pressure will not be the same owing to the surface tension between air and water at the point of contact.

- The meniscus which can be defined as surface curvature is formed by a liquid in a container (at the point of contact between the particles). This meniscus which has an edge on a soil particle will exert tension on that particle; this will only have significance if the meniscus is in contact with more than one particle.

- At higher saturation, small wettable objects floating on water tend to coalesce into rafts; in the case of bubbles, such rafts have many solid-like properties. Small wettable objects also tend to be attracted towards the edge of the meniscus. The gas bubbles have direct effect especially when it is in contact with more than one particle.

- The history of a soil is not clear whether it is on drying or on wetting path and the effect of hysteresis and anisotropy becomes greater.

- These forces are affected by contact angle. Contact angles describe the shape of a small drop of liquid in contact with a solid. The drop will spread out until the liquid cohesion is balanced by its adhesion to the solid. Contact angles are the most practical way to characterize surface energies of solids.

All the above forces will diminish to zero when full saturation is reached. The difference in pressure raises the question of what combination of pressure reduces the stress carried by the soil particles at their point of contact, while the tension will increase the inter-particle contact forces. It should be noted at this stage that the direct
effect of the surface tension has often been neglected, which is a gross error.

At a low degree of saturation, water is held at the point of contact between particles, while air occupies the remainder of the voids. The air is continuous, and may be at atmospheric pressure; the pressure in the water depends upon the curvature of the meniscus, and is less than the air pressure. The water is discontinuous, apart from a film of water wetting the surface of the particle (the adsorbed or diffuse layer), which allows slow movement of water and tends to keep the water pressure uniform. Changing the air pressure in such a soil will change the pressure acting on the soil particle in the same way as it does in a saturated or a dry soil. An increase in air pressure will also increase the pressure at the point of contact. If the total pressure is a gauge pressure, the effect of atmospheric pressure will already have been taken into account, and an increase in $u_a$ (also expressed as a gauge pressure) above atmospheric will reduce the effective stress. However, the air in this condition is not rigidly fixed; it is mobile within the soil, both by migration of bubbles that are small enough to pass through the gaps between particles, and by diffusion. The pressure in small bubbles will be higher than that in large bubbles, which will in

\[ \sigma' = \sigma - (u_a - u_{\text{atmosphere}}) \]

In most cases, the pore air pressure will be atmospheric, and the effective stress thus defined would be equal to the total stress. The only significant effect on shear strength would then come from the water at the point of contact. There are important characteristics of the pressure in this water, which make its effect very different from those of water pressure in a saturated soil:

- This water, which is called capillary water, is always in a state of tension.
- Its pressure is less than the air pressure, therefore, it results in pulling the particles together, whereas the water pressure in saturated soil may be more than or less than the atmospheric pressure.
- It is local, acting only over a limited area at the point of contact, whereas the water pressure in saturated soil acts everywhere; the effective stress concept can consider the loading at the boundaries rather than within the soil, because the pressure acts everywhere (a pore pressure will reduce the stresses before they even reach the soil); on the other hand, in the case of unsaturated soil, the situation at the boundaries will be different from that at the inter-particle contacts, and it will not be possible to predict what is being carried by inter-particle contacts without some consideration of what is happening within the soil.

- It is directional, the force is acting in a direction governed by the geometry and orientation of the contact; in a saturated soil there is no need to consider the direction of water pressure at all, because its effect can be completely accounted for by considering actions at the boundaries of the soil element.
- It is anisotropic but in saturated soil it is hydrostatic.

The effect is therefore equivalent to a force increasing the contact force between the particles.

These forces could be seen as if they are imparting cohesion to the soil, like a contact adhesive between the particles. The value of the cohesion across a plane would be dependent upon the resolved components of each of these contact forces in the direction orthogonal to the plane being considered. It might be argued that one effect would be to inhibit dilation of a soil, since the force acts to prevent particles from moving apart. In the simplest model of dilation, spherical particles slide over each other and do not need to break the contact to achieve dilation, so no effect would be seen. However, Oda and Kazama (1998) showed that dilation in shear bands in angular soil produces large voids and significant separation of particles, so the apparent cohesion would certainly have an effect in inhibiting dilation. To predict this effect a model is required for shear strength which is more sophisticated than the simple (saw tooth) model, but the effect will diminish to zero at the critical state when no more dilation is taking place. Its effect on shear strength could be modeled using any form of effective stress.

At high degree of saturation, the water may not be confined to the points of contact between particles, and may be discontinuous, existing as isolated bubbles. These bubbles would tend to be in contact with soil particles, but with no meniscus attached to a particle, and hence no tension is expected to be acting on a particle. The pressure acting on the particles will just be the water pressure; the shear strength will be determined by the effective stress. However, the air in this condition is mobile within the soil, both by migration of bubbles that are small enough to pass through the gaps between particles, and by diffusion. The pressure in small bubbles will be higher than that in large bubbles, which will in
Air will, therefore, tend to diffuse from high saturation zones towards low saturation zones; gravity will ensure that the low saturation zones are at the top of any body of soil. A soil with a high degree of saturation will therefore tend to separate into an almost full saturation zone, and a zone in which the degree of saturation is low. At the edge of a saturated zone, \((u_a-u_w)\) will be dependent upon the radius of curvature of the water surface in the spaces between the particles. The separation will, of course, take time, governed by the rate at which water can diffuse and the rate at which water can flow to replace the diffusing air; it will therefore be more rapid in coarse soils than in fine soils.

In summary, at high degrees of saturation a sample of soil will be divided into: zones which are almost completely saturated, in which shear strength is governed by \(\sigma' = \sigma - u_w\); and zones in which the degree of saturation is low. At low degrees of saturation, the shear strength is a function of the effective confining stress and pore water forces, which are themselves dependent upon the degree of saturation, surface tension, number of contacts per unit area and their directions, and particles characteristics.

### 3. A SIMPLE MODEL

The study of particle packing and porosity in particulate systems is of industrial importance in geotechnical engineering. The space filling properties of the particles tend to dominate the engineering behavior. Pore geometry and the pore interconnectivity control fluid transmission behavior; while the coordinate number, i.e. the number of inter-particle contacts, governs the strength behavior. Packing is influenced by many factors. These include particle size distribution, particle shape distribution (interlock effect) and the asperity scale surface roughness (friction effect).

A simple model of unsaturated soil allows some fundamental aspects to be examined in detail. A sample consisting entirely of single-sized spheres, at the loosest possible packing, will not model the compression behavior, but will give an indication of the shear strength. Adjacent particles occupy the concerns of cubes. A real sample is more likely to be in a close packing arrangement, but this is much more difficult to represent. Therefore, only loose packing will be discussed in this paper. The model is illustrated in Figure (1). The radius of spheres is denoted by \(R\). A simple solution for very low degrees of saturation (e.g. Kezdi, 1974) obtains a limiting value for the contact force between particles, but the model developed below allows the relationship between the degree of saturation and the increase in normal stress to be determined.

**Relationship between Degree of Saturation and Filling Angle**

Considering a cylinder bounded of total volume \(V_t\), the volume of water can be expressed by subtracting the volume of air outside the meniscus \(V_a\) and the volume of solid \(V_s\):

\[
V_w = V_t - V_s - V_a
\]

\[
V_t = \int_0^\delta \pi r^2 dy = \pi r^2 y |_0^\delta = \pi r^2 \delta
\]

Where \(r\) and \(\delta\) is defined in Figure (1).

The meniscus radius depends on the assumption about the shape of the meniscus; the curvature of any shape is given by the well-known formula that states that:

\[
r_m = \frac{1}{\rho} \frac{d^2x}{dy^2} \left(1 + \left(\frac{dx}{dy}\right)^2\right)^{3/2}
\]

Assuming that the meniscus is part of a circle, then

\[
X = \sqrt{r_m^2 - y^2}
\]

\[
\frac{dx}{dy} |_{y=\delta} = -\frac{y}{\sqrt{r_m^2 - y^2}} = -y \left[ \frac{2}{r_m - y} \right]^{1/2} = \frac{1}{r_m} \left[ \frac{2 - \delta}{r_m^2 - \delta^2} \right]^{3/2}
\]

\[
\frac{1}{\rho} \frac{d^2x}{dy^2} = -y \left[ \frac{2}{r_m - y} \right]^{1/2} - y \left[ \frac{2}{r_m - y} \right]^{3/2}
\]

\[
= \frac{-r_m^2}{\left[ \frac{2}{r_m - y} \right]^{3/2}}
\]
For small deflection, the following term:
\[
\frac{1}{(1 + \left( \frac{dx}{dy} \right)^2)^{3/2}}
\]
, can be neglected.

Then the meniscus radius can be found by using trial and error from the following equation:
\[
\frac{\delta}{r_m^2} + 2 \frac{r_m^2}{r_m^2 - y^2} + 3 \frac{r_m^2}{r_m^2 - y^2} \delta = \frac{-r_m^2}{1 - r_m^2} \frac{1}{\delta}
\]
\[
= \frac{2}{\delta - r_m^2 - y^2} \left[ \frac{r_m^2 - y^2}{\delta - r_m^2} - \frac{r_m^2}{\delta - r_m^2} \right]^{1/2}
\]

As illustrated in Figure (1), the half volume of water at the point of contact can be determined by integration:
\[
V_w = \pi \left[ \delta \left( r_1^2 + 2 r_1 r_m + 2 r_m^2 \right) - R \delta^2 \right] - \left( r_1 + r_m \right) \left( \frac{r_m^2 \sin^{-1} \left( \frac{\delta}{r_m} \right)}{r_m} + \sqrt{r_m^2 + \delta^2} \right)
\]
\[
\left. \frac{dx}{dy} \right|_{y=\delta} = \frac{-y}{\sqrt{r_m^2 - y^2}} \left. \delta \left[ \frac{2}{r_m^2 - \delta^2} \right]^{-1/2} \right|_{y=\delta}
\]

This expression is only valid for the filling angle up to 45 degrees for open packing arrangement. The degree of saturation can be calculated by using equation 6 as follows:
\[
S = \frac{6V_w}{8R^3 - 4\pi R^3 / 3}
\]

Again, considering a single sphere, the number of contact is N (e.g. 1.66-0.125N), but the volume of water associated with the sphere being considered is V_w as given in eqn.6, so the volume of water associated with each sphere is N.V_w. The volume of voids associated with each sphere is simply the void ratio multiplied by the volume of the sphere, so the degree of saturation can be written as follows:
\[
S = \frac{6V_w}{8R^3 - 4\pi R^3 / 3} = \frac{8(1.66 - e) V_w}{e(4\pi R^3 / 3)}
\]
\[
= \frac{1.66 - e \left[ 6V_w \right]}{e \left[ \pi R^3 \right]}
\]

The main problem in such a model is to estimate the contact angle. The contact angle is the angle at which a liquid/vapor interface meets the solid surface. The contact angle is specific for any given system and is determined by the interactions across the three interfaces. Most often the concept is illustrated with a small liquid droplet resting on a flat horizontal solid surface. Ideally, the droplet should be as small as possible because the force of gravity, for example, can actually change the above-mentioned angle.

Examination of glass beads under an optical microscope showed that the contact angle varies with the amount of water held between the points of contact (i.e. \( \beta \) is a function of \( \theta \)). These observations required the microscope slides and cover slips to be given a water repellent coating, so that a meniscus formed only against the particles being observed. The effect of this is limiting the curvature of the meniscus at a very low degree of saturation, hence; limiting the pressure difference across the meniscus. From figure (1) the angle \( \alpha \) can be written as follows:
\[
\tan \alpha = \frac{dx}{dy} \left|_{y=\delta} \right. = \frac{-y}{\sqrt{r_m^2 - y^2}} \left. \delta \left[ \frac{2}{r_m^2 - \delta^2} \right]^{-1/2} \right|_{y=\delta}
\]

Substituting \( \delta \) gives;
\[
\alpha = \tan^{-1} \left( \frac{R(1 - \cos \theta)}{\sqrt{r_m^2 - R^2(1 - \cos^2 \theta)}} \right)
\]

Substituting \( r_m \) in the above equation and rearranging it, the equation becomes:
\[
\alpha = \frac{R(1 - \cos \theta)}{\sqrt{R^2(1 - \cos^2 \theta) - R^2(1 - \cos^2 \theta)}}
\]
Equations 7, 9 and 10 can then be used to determine a relationship between \((u_a-u_w)\) and the degree of saturation. This is shown in Figures (2), (3) and (4) for three different particle sizes.

The suction calculated above is applied to the soil particle over the water at the point of contact, and acts to increase the contact force. The surface tension in the meniscus also increases the contact force. The total increase in force at a point of contact is, therefore:

\[
\Delta F = 2\pi T R \sin \theta + \pi R^2 \sin^2 \theta \left[ U_a - U_w \right] \quad (11)
\]

The increase in force at each point of contact will result in an increase in the normal stress carried out by the soil particles. The increase in stress on a plane passing through the contacts in the loose packing arrangement can easily be shown to be the same as the increase in the normal stress on other planes by resolving the forces. The stress increase \(\Delta\sigma\) will be equal to the increase in force from equation 11 divided by the area \((A = 4R)\).

\[
\Delta\sigma = \frac{2\pi T R \sin \theta + \pi R^2 \sin^2 \theta \left[ u_a - u_w \right]}{4R^2} \quad (12)
\]

For packing arrangements which have voids ratio smaller than the 0.90909 of the ideal loose packing, an adjustment must be made to take into account both the increased degree of saturation associated with a given water angle, and the increase in the number of contacts per unit area. Also, as the number of contacts per particle increases, the angle between adjacent contacts must reduce, lowering the maximum value of water angle for which the assumptions made above are valid.

This may be examined by considering a sphere intersected by a plane (the plane passing through the centre of the sphere is a special case; the following discussion applies to the general case). The sphere has a large number, \(N\), of evenly distrusted contacts, on each of which a normal force \(F\) is exerted; the effect of these forces will be equivalent to pressure. The resolved components of these forces normal to the plane will be equal to the same equivalent pressure applied to the area of the intersection between the sphere and the plane. The equivalent pressure on the surface of the sphere is \(NF/(4\pi R^2)\). For any orientation of the plane intersecting a large number of spheres, the plane will cut those spheres in various ways, but the proportion of the area of the plane
occupied by spheres will be $1/(1+e)$; the resulting increase in normal stress will be given by the equivalent pressure applied over this area, so:

$$\Delta \sigma = \frac{NF}{(1+e)(4\pi R^2)}$$  \hspace{1cm} \text{(13)}

Which is the same answer found above (eqn. 12); this expression is now assumed to be valid for the full range of void ratios encountered with single-sized spheres. It is then necessary to determine the relationship between co-ordination number $N$ and voids ratio. If the number of contact per unit volume in terms of void ratio $(N=8(1.66-e))$ is introduced into (eqn. 13), the equivalent pressure can be expressed as:

$$\Delta \sigma = \frac{8F(1.66-e)}{(1+e)(4\pi R^2)} = \frac{F(10-6e)}{(1+e)(3\pi R^2)}$$  \hspace{1cm} \text{(14)}

$F$ is determined from the water angle in the equations above; to relate $F$ and hence $\Delta \sigma$ to a degree of saturation, it is then necessary to determine the degree of saturation from the water angle $\theta$ for values of $e$ less than 0.909.

This increase in stress is shown in Figures (5), (6) and (7) for the particle sizes shown in Figures (2), (3) and (4), and will result in an increase in shear strength in accordance with the frictional properties of the material.

Taking the equations (6), (8), (11) and (14) together will allow the prediction of both the degree of saturation, $S$, and $\Delta \sigma$ for a given value of water angle, $\theta$, hence allowing the relationship between the two to be derived over a range of values of void ratio. Predicting $\Delta \sigma$ from $S$ requires a trial-and-error approach to determine the value of $\theta$ in order to give correct value of $S$. This is a simple task that can be conducted by using a worksheet (Excel).

**Filling Angle Limit**

The expression above is valid only when the water is ‘pendular’ at the point of contact between the particles, corresponding to a $\theta$ of 45° in open packing, and degree of saturation about 18%. At lower void ratios, the limiting value of $\theta$ reduces, but in a way that is not straightforward. For the percent study, it is proposed that $\theta_{lim}$ is taken to vary linearly from 45° at $e=0.909$ to 30° at $e=0.65$.

$$\theta_{lim} = 45 - \frac{15(0.909-e)}{0.909-0.65} = 45 - 58(0.909-e)$$  \hspace{1cm} \text{(15)}

At higher degree of saturation, microscopic observations suggest that soil is divided into zones; those that have some degree of saturation and others which are fully saturated. As the overall degree of saturation increases, the proportion of soil which is fully saturated also increases. Gravity will tend to make the saturated zone occupy the lower part of a sample, but in samples of fine grained soil it will take some time for this segregation to become complete. Published results for suction values in fine grained soils are usually laboratory values measured before segregation has taken place. The overall increase in effective stress due to the unsaturated zones will then be given by considering the value of $\Delta \sigma$ at $\theta_{lim}$, acting over a proportion of the cross-sectional area.

If the proportion of the soil volume which is unsaturated is $\rho$, then the overall saturation $S$ may be determined from $\rho$ and the saturation $S_u$ at $\theta_{lim}$ as follows:

$$S = \rho S_u + (1-\rho)(1-S_u)$$  \hspace{1cm} \text{(16)}

Hence, for a given $S$,

$$\rho = \frac{1-S}{1-S_u}$$  \hspace{1cm} \text{(17)}

Then the increase in effective stress is given by:

$$\Delta \sigma = \rho \Delta \sigma \theta_{lim}$$  \hspace{1cm} \text{(18)}

However, if there is some freedom in which shearing can take place, as the degree of saturation increases it will become progressively easier for shearing to avoid regions of low saturation, and the shear strength will approach that of the saturated value.

**Particle Packing Factor**

From the geometry shown in Figure (1-b), the particle packing factor is the fraction of volume in a unit volume that is occupied by particles. It is dimensionless. For practical purposes, the PPF is determined by assuming that particles are rigid spheres. It can be presented mathematically by:

$$PPF = N \frac{\text{Particle Volume}}{(\text{unit volume})},$$  \hspace{1cm} \text{(19)}

where $N$ is the coordinate number $(N=8(1.66-e))$. It can be mathematically calculated as one-component
particles (one type of particle) for close-packed structures.

As illustrated in figure (1-b), the body-centered cubic structure contains eight particles on each corner of the cube and one particle in the center. Each corner particle touches the center particle. A line that is drawn from one corner of the cube through the center and passes through to the other corner (radius of center particle and radius of the corner particle). By geometry, the length of the diagonal is $a = \sqrt{3}$. If the particle of the sample has the same radius $R$ (case 2). Therefore, the length of each side can be related to the radius of the particle by:

$$ a = \frac{4R}{\sqrt{3}} $$

However, the real case is that the particles have different radii. Let us assume the worst case (case 3 in figure 1-b) that the smaller size at the corner ($r$) and the biggest size at the center ($R$). The length of each side can be related to the radius of the particle by:

$$ a = \frac{2(R + r)}{\sqrt{3}} $$

Knowing this and the formula for the volume of a sphere, it becomes possible to calculate the PPF.

If the sample is mono-size, the PPF can be written as follows:

$$ PPF = N \frac{\text{Particle Volume}}{(\text{unit volume})} $$

$$ = \frac{N(4/3)\pi R^3}{\frac{4r}{\sqrt{3}}} \approx \frac{N}{3} = 2.67(1.66 - e) $$

Also, if the sample is mono-size the PPF can be written as follows:

$$ PPF = N \frac{\text{Particle Volume}}{(\text{unit volume})} $$

$$ = \frac{N(4/3)\pi R^3}{\frac{2(R + r)}{\sqrt{3}}} = 21.75(1.66 - e) \frac{R^3}{(R + r)^3} $$

Where the $R$ will be considered as the maximum radius in the sample and the $r$ will be considered as the minimum radius in the sample.

Then, the overall increase in effective stress stated in equation 18 will be given by:

$$ \Delta \sigma = \rho \left( PPF \right) \Delta \sigma \left. \right|_{lim} \quad \ldots(22) $$

Angular particle packs differ from circular or spherical particle packs in that the orientation of the particle influences the interlock between the fragments. Numerical modeling (Lin and Ng, 1997) shows that even ellipsoids behave quite differently from spheres during packing. In order to get a good approximation of the number of contact (Coordinate number), PPF concept should be used as illustrated in equation 22.

**Meniscus Length**

In fluid mechanics, the cheerios effect is the tendency for small wettable floating objects to attract one another. It is named for the breakfast cereal Cheerios and is due to surface tension and buoyancy. The same effect governs the behavior of bubbles on the surface of fizzy drinks.

Vella and Mahadevan (2005) discussed the cheerios effect and suggested that it may be useful in the study of self-assembly of small structures. They calculate the force between two spheres of density $\rho_s$ and radius $R$ with a floating distance $\ell$ apart in a liquid of density $\rho$ as:

$$ 2T\pi RB^{3/2}\Sigma K_1\left(\frac{\ell}{L_c}\right) $$

where $T$ is the surface tension, $K_1$ is a modified Bessel function of the first kind, $B = \rho gR^2 / T$ is the Bond number, and the factor $\Sigma$ is a non-dimensional factor in terms of the contact angle $\theta$ which can be written as follows:

$$ \Sigma = \frac{2\rho_s}{\rho} - \frac{1}{3} - \frac{\cos \theta}{2} + \frac{\cos^3 \theta}{6} $$

The meniscus length can be stated as follows:

$$ L_c = R/\sqrt{B} \quad \ldots(23) $$

The cheerios effect also refers to small objects that repel one another. Vella and Mahadevan (2005) showed that two small floating items with identical wetting properties can repel one another if the relative density of them is of opposite sign.
4. DISCUSSION

The model presented here demonstrates the direct effect of surface tension on the contact forces between particles, in addition to the indirect effect of the pressure difference between water and air which arises from surface tension and the curvature of the meniscus. It has been a common practice to neglect the direct effect of surface tension which is of the same order as the indirect effect of the pressure difference. This has been a serious error.

It is interesting to consider the effect of the assumption regarding the contact angle, which for this work is based upon optical microscopic observation of glass beads and mathematical derivation. It may be noted that the pressure difference will be far greater if the contact angle remains zero regardless of the degree of saturation; such pressure differences have not been observed (it may be noted that to reliably observe them would be very difficult), and it appears very likely that variations in the contact angle are the reason for this. The much higher difference results in a contact force which continues to increase as saturation drops to very small values, which is not correlated to with observations of shear strength. The influence of the contact angle on predictions is therefore very important, but it is increasingly difficult to be observed as particle size decreases. Contact angle can be expected to vary according to the nature of the soil particles and pore fluid. Predictions based upon the theory presented will therefore only be accurate under very carefully controlled conditions, but will give greater insight into the widely used empirical approaches based upon the use of various factors, or extensions of the critical state model.

Simple capillary rise tests on silty sand soil with narrow range of particle sizes (passing sieve 0.074 mm and retained on sieve 0.063) have been carried out to measure the capillary height. The average particle diameter is 0.069 mm. The capillary height, $h_c$, can be converted to matric suction using the well-known formula as follows:

$$U_a - U_w = h_c \gamma_w$$

...(24)

Figures (8) and (9) show the comparison between the predicted and the measured suction by using equations 5, 9, 10 and 21 for different void ratio 0.91 and 0.8. The comparison gives a good indication of the validity of the above equations.

5. CONCLUSIONS

The shear strength of soil at low degrees of saturation is increased by the effect of surface tension pulling particles together and so increasing the contact force. This increase is augmented by the effect of the pressure difference across the curved meniscus at the point of contact. The effect is strongly related to particle size. The effect can be related to the degree of saturation, if voids ratio is taken into account. The angle which the meniscus makes with the particle surface has an important influence, but is not easily observed. Any factor which alters the contact angle, such as the nature of the soil particle, surface contamination, or changes in the pore water, are likely to have a significant effect on the shear strength of soil at low saturation. These factors must therefore be taken into account before use is made of empirical methods for predicting shear strength.
Figure 1-a: Geometry of meniscus at the point of contact
(\(\phi\) is the angle between the tangent to the meniscus where it touches the sphere and the direction of the contact between the two spheres).

\[
\begin{align*}
\delta &= R(1 - \cos \theta) \\
r &= R \sin \theta
\end{align*}
\]

Tangent to the meniscus

\[
r_1 = R \sin \theta + r_m \cos \phi - r_m
\]

Case 2

\[
\beta + \alpha + \theta = \frac{\pi}{2}
\]

Case 3

Figure 1-b: case 1 the structure of pore in 3D, case 2 all the particles have the same size, case 3 the particles have different sizes.
Figure 2: The effect of saturation on pore water suction in coarse silt.

Figure 3: The effect of saturation on pore water suction in medium silt.

Figure 4: The effect of saturation on pore water suction in fine silt.
Figure 5: The effect of saturation on contact stress in coarse silt.

Figure 6: The effect of saturation on contact stress in medium silt.

Figure 7: The effect of saturation on contact stress in fine silt.
Figure 8: Comparison between predicted and measured suction.

Figure 9: Comparison between predicted and measured suction.
REFERENCES

Maaitah, O. 2004. Effect of Pore Geometry on Predicting the Shear Strength of Unsaturated Soil. Dirasat, Jordan University, 31 (1).