Dirasat, Engineering Sciences, Volume **32**, No. **1**, 2005

Behavior of Steel Plates Under Axial Compression and Their Effect on Column Strength

*Ahmed H. Al-Wathaf and Yasser M. Hunaiti **

ABSTRACT

An experimental study is conducted to investigate the behavior and postbuckling strength of plate elements in square and rectangular steel hollow sections having width-thickness ratios more than that in common rolled sections and their effect on the strength of columns. Two types of stub columns are tested under axial compression until failure; ordinary tubes and tubes reinforced by longitudinal stiffeners.

Based on the test results, two empirical compact equations for the effective section take into account the interaction between plate elements that are formulated to predict the postbuckling strength for both types. The interaction strength of columns having tube sections is predicted using the modified SSRC (Structural Stability Research Council) column strength equations to consider the effect of local buckling of plate elements. All the results are compared with the relevant formulas and the AISC specifications.

INTRODUCTION

Most structural steel members are composed of flat plate elements, which form flanges and webs of the cross sections. The performance of these members to carry loads depends mainly upon the properties of the members as a whole as well as the properties and the behavior of the plate elements.

Two cases of instability may occur in a steel column; overall (global) column instability and plate element (local) instability. Local buckling of plate elements in a steel column can cause premature failure of the entire section and reduce the overall strength.

The nature of manufacturing process of square and rectangular steel hollow sections requires forming these sections with lower thickness, which may affect their structural performance. Also, variations in structural characteristics introduced during the forming process of these sections may affect their strength and the behavior of their plate elements and, therefore, these matters must be verified.

* Civil Engineering Department, University of Jordan.
Received on 10/11/2003 and Accepted for Publication on 3/3/2005.

THEORETICAL ANALYSIS

Elastic Stability of Rectangular Plates under Uniaxial Compression

Euler formula for critical stress of a plate can be written in the following form (Bleich, 1952):

$$\sigma_{cr} = k\frac{\pi^2 E}{12(1-\nu^2)(b/t)^2} \qquad (1)$$

where:

k buckling stress coefficient.
E elastic modulus of elasticity.
ν Poisson's ratio.
b width of plate
t thickness of plate

Postbuckling Strength of Uniaxially Compressed Plates

When the magnitude of the critical stress in a plate is reached, slight buckling waves will appear very gradually, however the plate will not fail. It will continue to carry increasing load, sometimes a large multiple of that which causes the first barely perceptible waving, particularly when the width/thickness, *b/t,* ratio is large. This phenomenon is known as postbuckling strength and is of decisive importance for thin-walled metal structures.

A very important semi-empirical method of estimating the maximum strength of plates is the effective width concept. Only a fraction of the width is considered effective in resisting the applied compression. In a plate structure, use of the effective width leads to an effective cross-section consisting of portions meeting along a junction. It is near these junctions that the plates will begin to yield preceding failure. In postbuckling range, a load P that the plate carries can be expressed as:

$$P = \sigma_e b_e t \qquad (2)$$

where:

σ_e maximum stress at edges.

b_e effective width.

Euler formula, Eq.1, for critical stress can be written in the following form:

$$\frac{\sigma_{cr}}{\sigma_y} = \frac{1}{\lambda^2} \qquad (3)$$

Karman's equation for the maximum strength in terms of effective width of plate can be written as follows (Galambos, 1998; Usami and Fukumoto, 1982):

$$\frac{\sigma_u}{\sigma_y} = \frac{b_e}{b} = \frac{1}{\lambda} \quad \leq 1 \qquad (4)$$

Also, Winter's equation, can be expressed as:

$$\frac{\sigma_u}{\sigma_y} = \frac{b_e}{b} = \frac{1}{\lambda}(1 - 0.22\frac{1}{\lambda}) \quad \leq 1 \qquad (5)$$

Where

$$\lambda = \frac{b}{t}\sqrt{\frac{12\sigma_y(1-\nu^2)}{\pi^2 Ek}} \qquad (6)$$

For obtaining the effective section, A_e, for the whole rectangular hollow sections, Fig.1 shows the values of buckling coefficient, k, which should be used in Eq.6, which take into account the interaction between the plate elements (Galambos, 1998; Batista *et al*., 1987).

Stability of Rectangular Plates with Longitudinal Stiffeners

I-Antisymmetric mode of buckling

In this mode, the moment of inertia of the stiffener, I, is greater than or equal to I_o , (the moment of inertia of the effective cross-sectional area). Antisymmetric and symmetric mode of buckling are possible simultaneously when the moment of inertia of stiffener is equal to I_o, which means that the plate is adequately stiffened, and buckles between the stiffener and the edges. The coefficient of buckling for this mode can be determined as follows (Desmond *et al*., 1981):

$$(k_b)_{as.} = (b/b')^2 k_{b'} = 4k_{b'} \qquad (7)$$

where

$(k_b)_{as.}$ buckling coefficient which is expressed as a function of b.

b' $b/2$.

$k_{b'}$ buckling coefficient which is expressed as function of b'.

If we assumed simply supported unloaded edges of the plate, $k_b=16$.

II-Symmetric Mode of Buckling

In this mode, the moment of inertia of the stiffener, I, is smaller than or equal to I_o and the plate is considered partially stiffened. For this case when $0 \leq I/I_o \leq 1$, Desmond *et al*. (1981) suggested the following expression:

$$(k_b)_{sy} = (I/I_o)^{1/2}\left[(k_b)_{as} - (k_b)_{ns}\right] + (k_b)_{ns} \qquad (8)$$

where

$(k_b)_{sy}$ predicted buckling stress coefficient for partially stiffened plates.

$(k_b)_{as}$ buckling stresses coefficient for adequately stiffened=16 for simply supported unloaded edges.

$(k_b)_{ns}$ buckling stress coefficient for plate without stiffeners=4 for simply supported unloaded edges.

Interaction Between Local and Overall Buckling

The behavior of a column that exhibited local and overall buckling is very complex. The main difficulty lies in the non-linear behavior of thin-walled column, which results from the interaction of several instability modes; column flexural buckling, plate buckling, and flexural-torsional buckling.

The average stress on the total section of a column at failure is:

$$\sigma_{av} = \frac{P}{A} = \frac{A_e}{A}\sigma_e = Q\sigma_e \qquad (9)$$

where

Q strength reduction factor due to local buckling of plate elements.

σ_e average stress on the effective section at buckling.

Details of Tests

Test Specimens

The structural hollow steel tubes tested in this investigation are classified as cold-formed sections. Two different sections were prepared and tested, ordinary tubes (unstiffened tubes), and tubes reinforced by longitudinal stiffeners on all sides (stiffened tubes). More details of experimental program are presented by Al-Wathaf (2002).

The lengths of specimens were chosen to be sufficiently short to prevent overall buckling but long enough to permit local bucking of the individual component plates. The corners radii of sections were considerably small, therefore, their effects were neglected and the corners of the sections were treated as right-angle corners. Dimensions of the specimens are shown in Figs. 2 and 3 as well as in Tables (1 and 2). In these Tables, "S", "R", and "ST" refer to square tube, rectangular tube, and stiffener, respectively. The mechanical properties for specimens are shown in Table (3).

I-Unstiffened Tubes

The total number of this type is 12 specimens. Two specimens, A and B, for each cross section were tested to assure the precision of test results. The range of slenderness ratio, *b/t*, for plate elements was from 65.67 to 31.26 as shown in Table (1).

II-Stiffened Tubes

The total number of this type is 24 specimens divided into two groups. In group I, small stiffeners were used compared to the sectional area of the tubes whereas in group II, relatively large stiffeners were used. Two specimens, A and B, for each cross section were tested to assure the precision of test results. The description of these specimens is shown in Fig. 3 and Table (2). The stiffeners were attached to the tubes by welds on all sides. The dimensions of stiffeners were chosen such that local buckling of stiffeners themselves is prevented. Furthermore, the stiffeners dimensions were chosen so that their moments of inertia about their center are above and below values given by AISI (American Iron and Steel Institute) specifications, in order to allow for distortions of all modes. Three types of stiffeners were used (ST1, ST2, and ST3); dimensions of these stiffeners are shown in Fig. 3 and Table (2).

Test Rig

The stub columns were tested by incremental monotonic loading in a 2000 kN capacity M1000/RD universal testing machine. A control and measurement cell attached to the testing machine capable of providing data about load, displacement, and control mode was used to collect the experimental data. Computer software was incorporated to read and store all measurements including strain gauges and LVDTs (Linear Variable Displacement Transducers) readings.

Instrumentation

The stub column specimens were instrumented to measure loads, vertical displacement, deflections, and strains. Six specimens from Group I were chosen and tested with strain measurements. All strain gauges were placed at midheight of the specimens and distributed at corners and at the stiffener-plate junctions. Two strain gauges were also placed on stiffeners. The objective from using strain gauges in Group I is to check if the stiffeners of this group, which their sectional area is considered smaller compared to the section area of tubes than Group II, can carry stress effectively up to failure.

Results and Discussion

Postbuckling Strength and Stiffeners Efficiency

The ratio of the ultimate stress on the total area and the yield stress, F_u/F_y, against the slenderness parameter,λ , from test results and that predicted by Karman's and Winter's formulas (discontinuous lines), for a comparison, are shown in Figs.4 and 5 for unstiffened and stiffened tubes, respectively. Other comparisons for the postbuckling strength of sections for all cases of side stiffening (no stiffeners, stiffeners in Group I, and stiffeners in Group II) are shown in Fig.6, which clearly indicate the improvement in the postbuckling strength of tubes. Fig.6 represents the ratio of the load carried by tube sections alone for all cases using the average load of specimens A and B to the crushing load, $(P_u)_{tube}/P_y$, against the ratio of moment of inertia of the stiffeners to the forth order power of the tube thickness, I_s/t^4. It is observed from Fig.6 that the higher improvement in the postbuckling strengths occurs for tubes having higher width/thickness ratios.

The ratio of $(P_u)_{st}/(P_u)_{unst}$,as a percentage, is drawn versus the ratio of the sectional area of stiffeners and tubes in Fig.7 as well as the linear trend lines for Groups I and II. It is obviously observed from the slops of the two lines (Fig.7) that the strengths of specimens of Group I reach higher value even though the stiffeners were lighter. Moreover, no improvement in stiffening effect was obtained through the heavier stiffeners and the lighter stiffeners were sufficient to produce optimum effect and, therefore, the lighter stiffeners are more efficient. All the previous observations also can be shown in Fig. 6.

Proposed Effective Width Equations

Compact effective section formulas for unstiffened and stiffened tube sections based on the test results are derived which take into account the interaction between plate elements of the sections and yield the entire effective section directly.

I-Unstiffened Square and Rectangular Tubes

From regression analysis for test results, the compact effective section as follows:

$$\frac{A_e}{A} = \frac{0.714}{\lambda} \tag{10}$$

Substituting λ into Eq.10, using $\nu = 0.3$, the proposed equation can be expressed as follows:

$$\frac{A_e}{A} = \frac{0.68}{(b/t)}\sqrt{\frac{kE}{\sigma_y}} \quad \leq 1 \tag{11}$$

Prior to failure, the yield stress at the edges is replaced by σ_e and Eq.11 can be written as:

$$A_e = A\left(\frac{0.68}{(b/t)}\sqrt{\frac{kE}{\sigma_e}}\right) \tag{12}$$

where

b/t maximum width/thickness ratio of the plate in the cross section.

k buckling stress coefficient determined from Fig.1.

A special case can be considered when the tube is square and the widths of all plate elements are equal with $k=4.0$. For this case, Eqs. 11 and 12 can be written as:

$$\frac{b_e}{b} = \frac{1.36}{(b/t)}\sqrt{\frac{E}{\sigma_y}} \quad \leq 1 \tag{13}$$

$$b_e = 1.36t\sqrt{\frac{E}{\sigma_e}} \tag{14}$$

For a given yield stress the plate element remains fully effective up to failure, if the ratio *b/t* is below a limiting value. This limiting value, $(b/t)_{lim}$, can be obtained by setting ,in Eq.11, the ratio of A_e/A equal to 1.0. Therefore, Eq. 11 yields:

$$\left(\frac{b}{t}\right)_{\lim} = 0.68\sqrt{\frac{kE}{\sigma_y}} \tag{15}$$

II-Stiffened Square and Rectangular Tubes

The postbuckling strength measured for this type of stub columns is assigned for the tube section alone. From regression analysis for test results, the compact effective section as follows:

$$\frac{A_e}{A} = \frac{0.13}{\lambda} + 0.71 \tag{16}$$

Substituting λ into Eq.16, using $\nu = 0.3$, the proposed equation can be expressed as follows:

$$\frac{A_e}{A} = \frac{0.12}{(b/t)}\sqrt{\frac{k_b E}{\sigma_y}} + 0.71 \quad \leq 1 \tag{17}$$

Prior to failure, the yield stress at the edges is replaced by σ_e and Eq.17 can be written as:

$$A_e = A\left(\frac{0.12}{(b/t)}\sqrt{\frac{k_b E}{\sigma_e}} + 0.71\right) \tag{18}$$

where

b/t maximum width/thickness ratio of the total plate width, *b*, in cross-section.

k_b buckling stress coefficient of the total plate width, *b*, determined from Eqs.7 and 8.

The limiting value of fully effective plate can be obtained from Eq. 17 which yields:

$$\left(\frac{b}{t}\right)_{\lim} = 0.42\sqrt{\frac{k_b E}{\sigma_y}} \tag{19}$$

Characteristics of the Proposed Equations

I-Unstiffened square and rectangular tube sections

The characteristics of Eqs.11 and 12 can be summarized as follows:

- Their form is shorter than Winter's formula.
- Utilizing the interaction between the plate elements in rectangular sections where buckling coefficient will be more than 4.0 (for uniform section thickness), whereas the AISC (American Institute of Steel Construction) equation of effective width uses 4.0 for all cases.
- Computing the entire effective section directly, whereas in the AISC equation it needs to add the effective width for each side independently (AISC, 1993;AISC, 1980).

II-Stiffened Square and Rectangular Tube Sections

Eqs.17 and 18 have the same characteristics as the previous equations (11 and 12) and additional advantageous can be summarized as follows:

- They allow the designer to predict the postbuckling strength for tube sections having stiffeners that are not stipulated in the AISC specification (AISC, 1993).
- They allow the designer to predict the postbuckling strength for tube sections having lighter stiffeners less than that are recommended in the AISI specification (AISI, 1980).

Figs.4 and 5 represent the postbuckling strength curve predicted by proposed equations (continuous lines) and other curves plotted by Karman's and Winter's formulas as well as the test results for unstiffened and stiffened tubes, respectively.

Interaction of Local and Overall Flexural Buckling

The column strength equations presented by the AISC specifications, the ASD and the LRFD (AISC, 1993; AISC, 1980), were adjusted to take into account the reduction of stiffness due to local buckling. In order to predict the interaction strength for columns having the tested sections, Eqs.12 and 18, which give the effective section for unstiffened and stiffened tubes, respectively when the stress at the edges of plate elements is less than or equal to the yield stress, were used. More details are presented by Al-Wathaf (2002).

Figures 8, 9, 10 and 11 represent the ratio of the average stress to the yield stress, F_{av}/F_y, versus the slenderness parameter of columns, λ_c, for some sections exhibited local buckling such as S1, R1, S1-ST1, and R1-ST2, respectively. The reduction coefficient, $Q=A_e/A=F_{av}/F_y$, for the cross sectional area at zero length of column in which $F_{av}=F_u$ for the proposed strength (Eqs.11 and 17) is shown in Tables (4 and 5) for unstiffened and stiffened tubes, respectively. For rectangular sections, the proposed equations yield the factor Q directly, whereas in the AISC it needs to compute this factor for each side independently and add these together to get Q. Fig. 12 also shows a single column strength curve for all sections which is developed using a fictitious yield stress, QF_y, instead of the real material yield stress, F_y.

CONCLUSIONS

Postbuckling Strength of Plate Elements and Stiffener Efficiency:

From the experimental study and for the variables investigated, the following conclusions can be drawn:

1. For the tested stub columns, Winter's formula, in general, slightly overestimates the postbuckling strength of the plate elements. The ratio between the experimental postbuckling strength to that predicted by Winter's formula varies between 83% and 105% for unstiffened tubes, and between 80% and 100% for stiffened tubes.
2. The discrepancies between the proposed postbuckling strength and the test results are within an acceptable range which varied between –9.5% and 14.3% for unstiffened tubes and between -13.4% and 8.3% for stiffened tubes and, consequently, the proposed effective section equations may be used for the tested sections.
3. The proposed effective section equations take into account the interaction between plate elements and yield the total effective section directly.
4. Distinct increase in the postbuckling strength can be attained using longitudinal stiffeners in which some tested sections become entirely effective as rolled sections. The improvement in the postbuckling strength was relatively higher for sections having higher width/thickness ratios. From comparisons, stiffened tubes were capable of supporting loads between 161% and 108% of the unstiffened tubes.
5. The test results also show that the smaller stiffeners are more efficient in increasing the strength compared to their size as well as their acceptable sight on columns.

Interaction Strength of Tubular Columns

From the predicted interaction strength of tubular columns, the following conclusions can be drawn:

1. The proposed effective section equations yield the total strength reduction factor, Q, for the sections directly.
2. The interaction between local and overall flexural buckling begins at relatively lower stress for higher values of slenderness ratio of plates. From comparisons, the proposed interaction strengths for unstiffened tubular columns are relatively closer to the predicted strength by the AISC, ASD and LRFD, for sections with relatively lower slenderness ratios of plates.

ACKNOWLEDGMENT

The authors acknowledge the support presented by the Deanship of Academic Research / University of Jordan for the experimental work.

NOTATION

A cross sectional area.
A_e effective cross sectional area.
A_{stiff} cross sectional area of stiffener.
A_{tube} cross sectional area of tube.
b_e effective width.
F_{av} average compressive stress.
F_u ultimate compressive stress.
F_y specified yield stress.
I_s moment of inertia of stiffener around its centroid.
k buckling stress coefficient of plate with appropriate subscript.
r radius of gyration.
λ slenderness parameter of plate.
σ compressive stress.

REFERENCES

Al-Wathaf, A. 2002. *Behavior of Steel Plates Under Axial Compression and Their Effect on Column Strength*. M.Sc. Thesis. University of Jordan, Amman, Jordan.

American Institute of Steel Construction (AISC). 1980. *Specifications for the Design, Fabrication, and Erection of Structural Steel for Buildings*. Chicago.

American Institute of Steel Construction (AISC). 1993. *Load and Resistance Factor Specification for Structural Steel Buildings*. Chicago.

American Iron and Steel Institute (AISI). 1980. *Specifications for the Design of Cold-Formed Steel Structural Members*. Washington.

Batista, E., Rondal, J. and Maquoi, R. 1987. Column Stability of Cold-Formed U and C Sections. Proc. of the Int. Conference on Steel and Aluminum Structures. *Steel Structures, Advances, Design and Construction*. Cardiff, UK.

Bleich, F. 1952. *Buckling Strength of Metal Structures*.

McGraw-Hill Book Company, New York.

Desmond, T. P., Pekoz, T. and Winter, G. 1981. Intermediate Stiffeners for Thin-Walled Members. *J. of Structural Division, ASCE*, 107(4): 627-648.

Galambos, T. V. 1998. *Guide to Stability Design Criteria for Metal Structures*. 5th. edition. John Wiley and Sons Inc., New York.

Usami, T. and Fukumoto, Y. 1982. Local and Overall Buckling of Welded Box Columns. *J. of Structural Division , ASCE*, 108(3): 525-542.

تصرف الصفائح الفولاذية تحت تأثير الضغط المحوري وتأثيرها على قوة الأعمدة

*أحمد الوظّاف وياسر الحنيطي**

ملخـص

تمت دراسة مخبرية لتصرف الصفائح الفولاذية لما بعد الانبعاج وذلك من خلال دراسة أعمدة أنبوبية مربعة ومستطيلة المقطع، وقد تمت تقوية نصف العينات بصفائح تقوية طولية.

وبناء على النتائج المخبرية لهذه الصفائح تم اقتراح علاقات رياضية لانبعاج الصفائح وقوتها بعد الانبعاج، وأشارت النتائج إلى تقارب كبير مع العلاقات المقترحة في المواصفات الأمريكية.

* قسم الهندسة المدنية، كلية الهندسة، الجامعة الأردنية. تاريخ استلام البحث 2003/11/10، وتاريخ قبوله 2005/3/3.