Stress Distribution of Two Closely Separated Single Footings on Infinite Soil Media

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ABSTRACT

The effect of two symmetric linearly varied line loads, placed adjacent to each other on a semi-infinite elastic continuum is studied. Five cases of loading are studied to include both normal load and varied moment effect. Non-dimensional curves are produced to indicate the variation along both horizontal and vertical sections of the continuum for: vertical deflection, normal stress, horizontal stress, and shear stress. The study is carried out to observe the effect of complete separation between adjacent parts of the structure, at an expansion joint, on soil stresses. For the uniformly applied load, the footing separation has no effect on the soil action. For other cases, the effect is within a square zone of twice the width of the footing. Therefore, extending the expansion joint vertically through the footing will produce unnecessary local high stresses.

KEYWORDS: Footings, Elasticity, Stress Analysis, Concrete Structure, Finite Element Analysis.

1. INTRODUCTION

Structures are subjected to many types of movements. Some of these are time dependent, e.g., creep, shrinkage and consolidation; others are not, e.g., temperature induced movement, and instantaneous elastic settlement. All of these movements depend mainly on building materials, type of construction, loading, and type of bearing strata.

Vertical joints are usually provided for two groups of movements, namely, differential settlement due to load/soil changes, and horizontal movement due to thermal/shrinkage expansion. Expansion joints are defined by ACI Committee 116 (1988) and Meyer (1996), as: *separation between adjoining parts of a structure provided to allow small relative movements such as those caused by temperature changes*. The Building Code Requirements for Structural Concrete (1999) defines an isolation joint as: *separation between adjoining parts of a concrete structure usually a vertical plane, at a designed location such as to interfere least with the performance of the structure, yet such as to allow relative movement in three directions and avoid formation of cracks elsewhere in the concrete and through which all or part of the bonded reinforcement is interrupted*. An ambiguity of this statement comes at the bottom of this separation where the foundation exists. Fintel (1997) defines a movement joint (functional joint) as: *Movement joints serve restraints which would otherwise occur as a result of differences in deformation of adjacent parts. In some cases, such joints are interposed between the structure and its foundation, since the superstructure deforms due to the external load and to temperature variation while the foundation remains immovably secured in ground*. This statement softly touched the expansion joint problem. Detailing manuals, such as ACI Detailing Manual (1980) and Boughton (1980) are not helpful in clearing such ambiguity. This ambiguity is encountered among practicing engineers and their judgment is solely based on previous experience.

In large structures the horizontal force at the bottom of the structure due to thermal expansion is resisted by several factors, including: tie beams at footing level, soil embedment effect on thermal insulation, and soil resistance. If the adjacent footing is not separated at the joint, an additional resistance against horizontal movement is made through the action between the two adjacent footings.

Research Significance

A discussion is made on the importance of extending the expansion joint through the foundation and its effect on the distribution of the stresses and displacements on the soil below the foundation. It is assumed that the vertical expansion joint creates two adjacent columns subjected to symmetric load and moment. This assumption, although idealizes the problem, is used throughout this research. If the joint is carried through the footings, then each footing is subjected to an eccentric load.

The distribution of stresses and displacements is studied. Non-dimensional charts are produced for stresses and displacements of two closely spaced line loads length of each is L units and load intensity varying linearly in a symmetric way from the case of a uniform load to the case of linearly varying load with zero load intensity at the far edge. The average load intensity is q for all cases.

Work Procedure

An idealized semi-infinite elastic continuum is adopted. Such idealization was followed by other researchers. Mete (1990) studied an inhomogeneous elastic half space. Dempsey and Li (1989) discussed a rigid rectangular footing on an elastic layer. This continuum is subjected to two closely spaced linearly varied loads of average intensity q units over a length of L units as shown in Fig.1.

The value of m, as shown in Fig.1, is varied from $m = 0$ for a uniformly distributed load to $m = 0.25, 0.5, 0.75,$ and 1. The value $m = 1$ expresses zero stress at the far end. For the uniformly distributed load case, a closed form mathematical solution is available by Young and Budyans (2001) as follows:

A closed form solution for the displacement just below the load is given by the following equations[1]:

$$y_1 = \frac{qL}{\pi \times E}\left[2\left(1 + \frac{X_1}{L} \times \ln\left(\frac{n}{1 + \frac{X_1}{L}}\right)\right.\right.$$
$$\left.\left. - \frac{X_1}{L} \times \ln\left(\frac{n}{\frac{X_1}{L}}\right)\right) + (1 - \upsilon)\right] \quad \text{...(1)}$$

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[1] The terms used in the equations are defined in Appendix I: Notations.

$$y_2 = \frac{qL}{\pi \times E}\left[2\left(1 - \frac{X_2}{L} \times \ln\left(\frac{n}{1 - \frac{X_2}{L}}\right)\right.\right.$$
$$\left.\left. + \frac{X_2}{L} \times \ln\left(\frac{n}{\frac{X_2}{L}}\right)\right) + (1 - \upsilon)\right] \quad \text{...(2)}$$

The stresses below a uniformly distributed load over elastic media is given by Prescott (1961): -

$$\sigma_1 = -\frac{q}{\pi}\left[(\alpha - \beta) + \sin(\alpha - \beta)\cos(\alpha + \beta)\right] \quad \text{...(3)}$$

$$\sigma_2 = -\frac{q}{\pi}\left[(\alpha - \beta) - \sin(\alpha - \beta)\cos(\alpha + \beta)\right] \quad \text{...(4)}$$

$$\tau = \frac{q}{\pi}\left[\sin(\alpha - \beta)\sin(\alpha + \beta)\right] \quad \text{...(5)}$$

Since linear elasticity and small deformation are assumed, the method of superposition is used to evaluate the effect of two adjacent symmetric footings.

In addition to the closed form solution given above, a finite element solution is carried out. Analysis is made using a Staad (Research Engineers) program. The mesh size adopted is of $24 \times 16 = 384$ square elements of dimension L used on half of the semi-infinite plate with L as half the length of the footing and a thickness of one unit. The loading is composed of two line loads each over a span L units which represents half the length of one footing. The dimension of each element is $\frac{L}{4} \times \frac{L}{4}$ units. The load was linearly varied to represent five cases of loading: $m = 0.25, 0.5, 0.75,$ and 1. The results obtained for the case of uniform load are compared for both the mathematical solution and finite element solution. The results recorded in tables (1, 2, and 3) show a close comparison.

Such close comparison between the closed form mathematical solution and the finite element for the case of uniform load ($m = 0$), as recorded in tables (1, 2, and 3), indicates the convergence of the finite element solution to the right answer in the other cases ($m = 0.25, 0.5, 0.75,$ and 1).

Curves for displacements and stresses are produced at various horizontal and vertical locations of the structure. These curves include vertical displacement, normal stress, horizontal stress and shear stress for the five cases of loading, as described before. The results of the same

action, for the five cases of loading, are superimposed over each other to indicate the variation of each action due to each case of loading. It is to be noted that all curves are plotted in a dimensionless form with the multiplier for each as shown on the abscissa and ordinate of the curve.

Discussion of Results

The curves for $m = 0$ represent the case of no vertical joint through the footing. Difference between $m = 0$ and any other value indicates the effect of inducing a vertical joint on any specific action. Stress was made on the following actions: vertical displacement, normal stress, and horizontal stress. Other effects, although mentioned yet it was considered of less importance.

Vertical Displacement

Figs. 4(a)-4(g) show that the maximum vertical displacement for the planes $Y = 0$, L, and $2L$. This vertical displacement is also traced at the planes $X = 0$, $0.5L$, L, and $2L$. This displacement is magnified at $m = 1$ in an almost linear way to 1.25 the value at $m = 0$. The zone primary affected by the applied moment is bounded by $X = 2L$ and $Y = 1.5L$.

Normal, Horizontal, and Shear Stresses

Figs. 5(a) and 5(b) show that at $Y = 0.125L$ the maximum normal stress is magnified by 1.7 at $m = 1$ compared to the value at $m = 0$. The zone primary affected by the applied moment is bounded by $X = 1.5L$ and $Y = 2L$.

The maximum horizontal stress is given less importance to economize on curves. However, it should be mentioned that it is also magnified by 1.6 at $m = 1$ compared to the value at $m = 0$. The zone primary affected by the applied moment is bounded by $X = 1.5L$ and $Y = 2L$.

The horizontal shear stress is also given less importance to economize on curves. It was noted that its maximum value is at $Y = 0.625L$ and is magnified by 1.35 at $m = 1$ compared to the value at $m = 0$. The zone primary affected by the applied moment is bounded by $X = 1.75L$ and $Y = 2L$.

CONCLUSION

The separation between two adjacent structures due to thermal effect is discussed. At the foundation level, the lower parts of both structures are embedded in soil, thus forming thermal insulation and resistance to thermal movement, hence the necessity of the separation of the structure at the foundation level is investigated. A model of two symmetric variable line loads length of each equals L on a semi-infinite elastic continuum was investigated. These line loads are varied in five stages from a uniform load to a zero pressure load at the far edge of the applied load. Non-dimensional curves are plotted for the different actions below the loads, namely: vertical deflection, normal stress, horizontal stress, and horizontal shear stress. For the case of a uniform load, the separation has no effect on the different actions within the continuum. For other load cases, the effect of all actions was magnified. The vertical displacement was magnified by 1.25, the normal stress by 1.7, the horizontal stress by 1.7 and the shear stress by 1.35. Beyond $X = 2L$ and $Y = 2L$, the effect of the separation decayed.

Therefore, it is recommended not to make a separation joint on a foundation at a vertical expansion joint unless it is necessary due to construction limitations. If such separation is avoided, the load distribution on the foundation would be less critical and the deflection and stresses on the structures are relieved.

ACKNOWLEDGMENT

Appendix I: Notations

α, β :   Angle in radians measured clockwise with the vertical as shown in Fig. 3.
E :   Young's modulus of elasticity.
L :   Half length of the footing of unit width.
m :   Load multiplier to indicate the variation of line load over the span.

n	:	Length multiplier to evaluate the distance of the reference point of measurement from the left edge of the footing (taken equal to 5).
O_1	:	Exterior point of measurement.
O_2	:	Interior point of measurement.
q	:	Average load intensity.
R	:	Reference point of measurement.
σ_1	:	Stress in the vertical direction.
σ_2	:	Stress in the horizontal direction.
τ	:	Horizontal shear stress.
ν	:	Poisson's ratio.
X_1	:	Distance of an exterior point measured from the left edge of the footing.
X_2	:	Distance of an interior point measured from the left edge of the footing.
Y_1	:	Deflection of an exterior point.
Y_2	:	Deflection of an interior point.

Appendix II: Tables

**Table 1. Deflection at top of an infinite elastic media with two uniformly distributed line loads.
Width of each equals _L_. _X_ = 0 at center line (Multiplier $qL / \pi \times E$).**

X / L	0	0.25	0.5	0.75	1	1.25	1.5	2	2.5	3
Math Solution	8.038	7.906	7.515	6.761	4.819	3.704	2.763	1.446	0.484	-0.28
FE Solution	7.952	7.834	7.459	6.772	5.318	3.817	2.988	1.891	1.202	0.744

**Table 2. Normal stress distribution due to two uniformly distributed vertical loads of intensity _q_.
Width of each load _L_ at a depth of 0.375_L_. _X_ = 0 at center line (Multiplier _q_).**

X / L	0.125	0.375	0.625	0.875	1.125	1.375	1.625	1.875	2.125	2.375
Math Solution	0.979	0.964	0.906	0.696	0.301	0.09	0.031	0.013	0.007	0.004
FE Solution	0.981	0.966	0.883	0.656	0.344	0.146	0.059	0.014	0.008	0.004

Table 3. Horizontal shear stress distribution due to two uniformly distributed vertical loads of intensity _q_. Width of each load _L_ at a depth of 0.375_L_. _X_ = 0 at center line (Multiplier $q \times 10^{-3}$).

X / L	0.125	0.375	0.625	0.875	1.125	1.375	1.625	1.875	2.125	2.375
Math Solution	17.54	62.16	142.92	273.96	276.59	151.26	77.81	44.02	27.28	18.13
FE Solution	15.72	57.8	149.2	259.7	260.97	152.4	64.8	30.5	14.1	4.81

Appendix III: Figures

- 61 -

- 62 -

- 63 -

- 64 -

- 65 -

- 66 -

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