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Failure Investigation of Reinforced Concrete Columns Exposed to Fire

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ABSTRACT

In this work, the probability of failure with time of reinforced concrete columns subjected to a high temperature is investigated. This study is of great concern, because under the effect of high temperature, the material properties of steel reinforced concrete degrade drastically. Different columns are taken under consideration by varying a fixed s et of parameters; these parameters include: the cross-section of the column, arrangement of the steel bars, diameter of the steel bars, distance of steel-bar center from column center, number of bars used in one column, reinforcing ratio (As/Ac%), surface emissivity, and fire temperature. The technique of finite element analysis was used to calculate the temperature distribution with time over the column's cross-sectional area using suitable computing machines. Rising temperature in the column leads to a decrease in the load-carrying capacity, which leads to a probable collapse of the building at a critical strain of ε = 0.3%. Out of the several cases which were deeply investigated, it was found that the time range needs to cause failure in column is between 140 to 191 minutes depending on the pre-mentioned study parameters.

Keywords: Column Load Capacity, Convection-Conduction-Radiation Heat Transfer, Finite Element Method, Fire in Buildings, Fire Resistance, Reinforcing Concrete Column

1. INTRODUCTION

Building fires often lead to the most painful tragedies, in which human lives and valuable property and assets are lost in a relatively short time. It is therefore essential to attempt to prolong the time before the building collapses. In other words, to force the building to take a relatively longer time before it collapses or falls. The time required for a building to collapse during a fire is an indication of the building's ability to resist failure due to thermal effects. In light of the fact that columns are the building's main support, delaying a column's failure directly leads to delaying the collapse of the building itself. Therefore, delaying of a columns' failure is a reliable factor for improving the building's fire resistance.

High temperature has a more drastic effect on load bearing columns than the effect of mechanical forces themselves. The effect of high temperature on structures comes in two folds; it drastically degrades the material properties, and secondly, as a result of the degraded material properties, decreases the column's load-carrying capacity. This fact has been seen by two major world fire accidents: the Chernobyl nuclear plant accident and the world-trade center accident; both melt down and collapsed entirely. Collapse of a column finally occurs when the column's load-carrying capacity drops below the available load on the column. This is also known as the fire resistance of the column.

Fire resistance of a reinforced concrete column depends primarily on the type of aggregate, dimensions of various parts comprising the column, temperature distribution over the column's cross section with time, and the cover of the concrete over the reinforcement. A column's fire resistance is defined as the time of acting effective fire before the column fails (Reynolds *et al.*, 2008). Furthermore, with increasing fire temperature, the column's temperature will increase and thus the column carrying capacity will decrease.

In this research the temperature rise in the column and temperature distribution over the column's cross section

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due to uniform fire temperature surrounding it is investigated to determine the time elapsed before column's failure. Failure time for a column as function of parameters such as fire temperature T_f, reinforcing ratio A_s/A_c, steel bar arrangement within the column cross section, types of steel bars, column-surface emissivity, thermal conductivity, and thermal expansion coefficients of steel and concrete are investigated.

Evaluating of the temperature distribution and the normal stress over the column-cross section with reference to time was analyzed and displayed. Numerical simulations were carried out using the finite element method.

2. Material Models

Steel and concrete work well together in reinforced concrete structures. The advantages of each material seem to compensate for the disadvantages of the other. For example, the great shortcoming of concrete is its lack for tensile strength, but tensile strength is one of the great advantages of steel. A reinforcing bar has tensile strength approximately 2 orders of magnitude greater than that of concrete.

Steel and concrete bond together tremendously well, so that there is no slippage between the two materials, and thus can be modeled as being "perfectly" bonded at the concrete-steel interface. This excellent bond between the two materials is due to the chemical adhesion between them. While steel is typically prone to corrosion, the concrete surrounding it provides it with excellent protection. Furthermore, concrete and steel work very well together in relation to temperature changes due to the fact that their coefficients of thermal expansion are close, (McCormac, 2008, Kulkarni, 1998).

The compressive strength of concrete, σ_c, is determined by testing 28-day-old 6 x 12 in, concrete cylinder at a specified rate of rate of loading. Furthermore, according to the American Concrete Institution (ACI) code 8.5.1, the effective modulus E_c is calculated through the following relationship as expressed in the ACI Standard 318-99:

$$E_c = \rho_c (1.5)(0.043)\sqrt{\sigma_{28,con}} = 0.0645\rho_c \sqrt{\sigma_{28,con}}. \qquad (1)$$

For ordinary applications, a value of $\sigma_{28,con}$ = 20.685 MPa is used, while for pre-stressed constructions a value of $\sigma_{28,con}$ = 34.475 MPa is quite common. It is quite feasible to increase $\sigma_{28,con}$ from 20.685 MPa to 34.475 MPa concrete without requiring excessive amounts of labor or cement. The approximate increase in cost for such a strength increase is approximately 15 - 20%.

The stress-strain curves shown in Figure 1 represent the results obtained from compression tests of sets of 28-day-old standard cylinders of varying strength, (McCormac 2008). These curves provide several significant points:

- Concrete behavior is almost nonlinear, especially at higher stresses. This nonlinearity causes some problems in the structural analysis of concrete structures.
- It can be seen that regardless of strength and stiffness, all of the concrete types reach their ultimate strengths at a strain of about 0.2%
- Concrete does not have a definite yield point, rather, the curves run smoothly on to the point of rupture at a strain range of: 0.3% $\leq \varepsilon \leq$ 0.4%. In this paper it is assumed that the concrete fails at ε = 0.3%
- It should be further noticed that the weaker grades of concrete are less brittleness than the stronger ones, (McCormac, 2008).

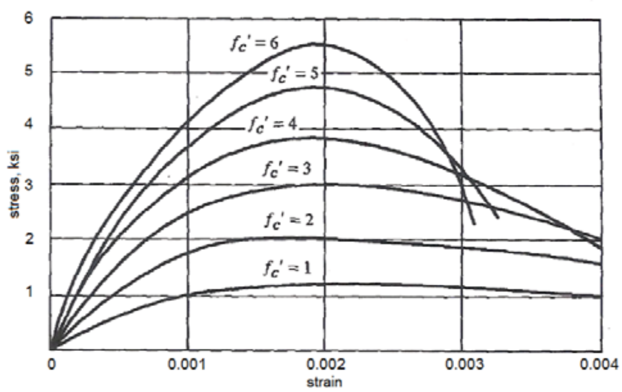

Figure 1: Typical Concrete Stress-Strain Curves (1 ksi = 6895 kPa), (McCormac 2008)

The effects of temperature on the constitutive relations of the column's materials are of great importance when generating a finite element model to measure the fire response of a concrete structure. These properties include, but are not limited to, thermal conductivity, specific heat capacity and thermal coefficient of expansion. For this model, the relationships for these parameters will be used in accordance with the results derived by (Kodur *et al.,* 2004) for high-strength concrete. To simulate the change in elastic modulus, a model proposed by (Xiao *et al.,* 2004) will be used and the equation for the specific heat of concrete will be based on a model described in the Eurocode standard EN1992-1-2:2004. The models state that:

$$c_c = \begin{cases} 900, & \text{for } 20°\text{C} \le \text{T} \le 100°\text{C} \\ 900 + (T - 100), & \text{for } 100°\text{C} \le \text{T} \le 200°\text{C} \\ 900 + -0.5 * T, & \text{for } 200°\text{C} < \text{T} \le 400°\text{C} \\ 1100, & \text{for } 400°\text{C} < \text{T} \le 1200°\text{C} \end{cases}, \quad (2)$$

$$k_c = \begin{cases} 0.85 * (2.0 - 0.0013 * T), & \text{for } 20°\text{C} \le \text{T} \le 300°\text{C} \\ 0.85 * (2.21 - 0.002 * T), & \text{for } \text{T} > 300°\text{C} \end{cases}, \quad (3)$$

$$\alpha_c = (0.008T + 6) * 10^{-6}, \quad (4)$$

$$\frac{E_{c,T}}{E_c} = \begin{cases} 1.0 - 0.0015 * T, & \text{for } 20°\text{C} \le \text{T} \le 200°\text{C} \\ 0.87 - 0.00084 * T & \text{for } 200°\text{C} \le \text{T} \le 700°\text{C}, \\ 0.28 & \text{for } \text{T} \ge 700°\text{C} \end{cases} \quad (5)$$

where,
C_c = Specific heat of concrete at temperature T
K_c = Thermal conductivity of concrete at temperature T
E_c = Modulus of elasticity of concrete at 20°C
$E_{c,T}$ = Modulus of elasticity of concrete at temperature T

It should be noted, for clarification, that typical ASCE concrete models, as well as the concrete models put forth by (Kodur *et al.,* 2004) are expressed as a function of aggregate, temperature and grade. For this model, we are assuming a high-strength concrete with a carbonate-type aggregate.

For the steel rebar, the model will assume that the steel is structural steel as defined by the ASCE. The models for the thermal and mechanical properties, as a function of temperature, will be based on the ASCE models as put forth by (Kodur *et al.,* 2004). The model states that:

$$\rho_s c_s = \begin{cases} (0.004T + 3.3) \times 10^6, & \text{for } 20°\text{C} \le \text{T} \le 650°\text{C} \\ (0.068T - 38.3) \times 10^6, & \text{for } 650°\text{C} \le \text{T} \le 725°\text{C} \\ (-0.086T + 73.35) \times 10^6, & \text{for } 725°\text{C} < \text{T} \le 800°\text{C} \\ 4.55 \times 10^6, & \text{for } \text{T} > 800°\text{C} \end{cases}, \quad (6)$$

$$k_s = \begin{cases} -0.022T + 48, & \text{for } 20°\text{C} \le \text{T} \le 900°\text{C} \\ 28.2, & \text{for } \text{T} > 900°\text{C} \end{cases}, \quad (7)$$

$$\alpha_s = \begin{cases} (0.004T + 12) \times 10^6, & \text{for } 20°\text{C} \le \text{T} \le 1000°\text{C} \\ 16 \times 10^6, & \text{for } \text{T} > 1000°\text{C} \end{cases}, \quad (8)$$

$$\frac{E_{s,T}}{E_s} = \begin{cases} 1.0 + \dfrac{T}{2{,}000 \ln\left(\dfrac{T}{1{,}100}\right)}, & \text{for } 20°\text{C} \le \text{T} \le 600°\text{C} \\ \dfrac{690 - 0.69T}{T - 53.5} & \text{for } \text{T} \ge 600°\text{C} \end{cases}, \quad (9)$$

where,
C_s = Specific heat of steel at temperature T
K_s = Thermal conductivity of steel at temperature T
E_s = Modulus of elasticity of steel at 20°C
$E_{s,T}$ = Modulus of elasticity of steel at temperature T

3. Model of Reinforced Concrete Columns

Plain concrete columns cannot support much load and therefore are not used as building columns. The additional of longitudinal bars greatly increase their load carrying capacity while further strength increase may be achieved by providing lateral restraints for those longitudinal bars, this is due to the fact that under compressive loads, columns tend not only to shorten lengthwise but also to expand laterally. At failure, the theoretical ultimate load or nominal load, P_n, of a short, axially loaded, unsupported column is accurately determined by the following expression (BS EN 1992-1-2: 2004):

$$P_n = 0.85\sigma_{y,c} A_c + \sigma_{y,s} A_s = 0.85\sigma_{y,c}(A_g - A_s) + \sigma_{y,s} A_s. \quad (10)$$

The factor 0.85 accounts for the fact that concrete poured into large forms does not exhibit identical properties to concrete poured into test cylinders used to establish the stress-strain curve, (McCormac, 2008).

In order to define a failure criterion, let the load carrying capacity of a column be the maximum design load that can be carried or resisted by the column. The critical load carrying capacity of the column is the load by which the column starts collapsing or failing.

As a criterion for failure, the column was assumed to fail when the strain in any concrete element reaches ε_f = 0.3%. A related static failure criterion may be assumed when the load-carrying capacity of column is equal or greater than the critical load-carrying capacity for the column.

For the FE model, a representative volume element, RVE, Figure 2, is a quarter section of the column which reduced computational time by taking advantage of the columns' symmetry. In order to further reduce the computational effort, it can be noticed that for a long column completely exposed to the fire, the problem reduces to a 2D plane strain problem. This can be justified since the column will experience little to no axial deformation. Adding to this the effect that the slenderness ratio (λ= 2R/L) is larger than λ_{crit} by which buckling occurs. It is assumed that buckling will not be the case in the presented analyses.

The FE model was compiled and run using ANSYS APDL. The concrete and steel elements were modeled using PLANE223 element which is a 2D 8-node coupled field element, see Figure 3.

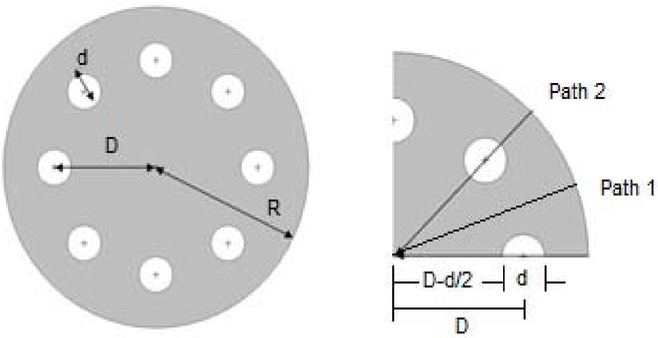

Figure 2: (a) Cross Section (b) RVE of Column

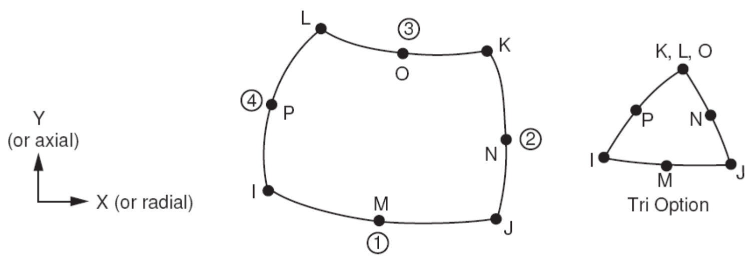

Figure 3: Plane223 Geometry

This element was chosen to limit the coupling to a structural-thermal coupling. This allows the model to include displacements as well as temperature as degrees of freedom. It can be seen from the model in Figure 4, the mesh was made finer in the area surrounding the steel rebar. This was done in order to accurately capture the temperature distribution around the rebar as well as the resulting stress field. The criterion for determining convergence adopted in this work is a comparison of strain at several locations of the concrete region. With an increasing number of divisions the value of the strain converges to a solution with decreasing error and the results for the taken cases are indiscernible.

Furthermore, with the increase in the number of divisions, the percent error in the strain is reduced according to the relation:

$$\%\ \text{error} = |\,\varepsilon_{\text{Ii}}^{n+1} - \varepsilon_{\text{Ii}}^{n}\,| / \varepsilon_{\text{Ii}}^{n+1} \times 100, \qquad (11)$$

where ε_I is the max principal strain evaluated at the ith strain for the nth increment in mesh refinement. Based upon the results from the strain in the concrete, an appropriate mesh density with acceptable accuracy for this system is Case 4 (as listed in Table 1). The max principal strain is used as a measure in this mesh convergence study since it is the primary indication of failure as indicated by the failure criteria previously discussed.

For purpose of studying the effect of fire on reinforced concrete columns, several cases have been designed and simulated, and then the data was processed. These cases include variations to the shape and dimensions of the column's cross section, arrangement of the steel beams, and the steel/concrete ratio. Among the many parameters that affect the heat transfer and the temperature distribution within the column cross section, the following were considered: the effect of the fire's temperature, the radius of the rebar array, the radius of the rebar, and the effect of the emissivity of the concrete. Table 1 summarizes the different cases investigated in this work.

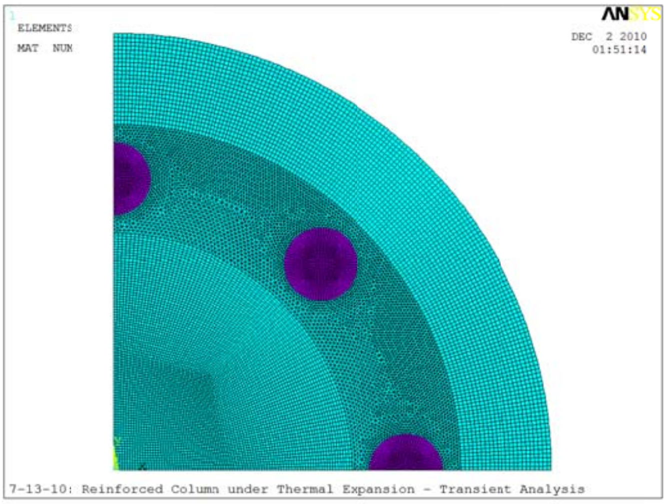

Figure 4: Finite Element Mesh of RVE

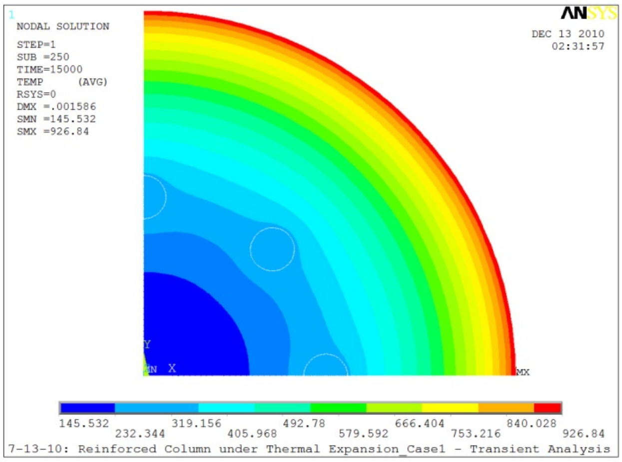

Figure 5: Temperature Distribution Through Cross-Section of Column

4. Results

Once the simulations were run, the results were compiled and plotted in order to graphically show the effects of temperature on the failure of the column. Along with temperature distribution, as shown in Figure 5, the effects of temperature on the stress and strain in the column were also analyzed.

Figure 6 shows the temperature distribution through path 1 for case 1. It is evident that as we move away from the column's center toward the outside surface, i.e., we come closer to the heat source, the column's temperature increases. This observation can be explained by Eq. (5). Furthermore, it is can be observed that the temperature variation with time in the inner region is significantly smaller than in the outer region. From Figure 6 we notice that $\Delta T(x = 0) \approx 115°C$ while $\Delta T(x = R) \approx 640°C$.

This difference is most likely due to the low thermal conductivity of concrete as can be deduced from Eq. 3. In addition this implies that as time increases, the temperature at each point on the curve shifts up, a phenomenon that is due to heating the column's body and results in an increase in the stored energy in the column accompanied with increasing specific heats.

Figure 7 illustrates the variation of strain in the concrete versus the distance along path 1 for case 1. During any given period of time, the column temperature has increased. Furthermore, the strain will increase with temperature according to Eq. 10. Additionally, the stress will vary in the same manner as the strain because of the direct relationship between them.

Table 1: Summary of Cases for Studying the Effect of Fire on Reinforced Concrete Columns

Case No.	Beam Geometry [m]	Number of Bars	As/At (%)	Fire Temperature [K]	Failure Time [min]
1	R = 0.2; d = 0.02865; D = 0.1	8	4.1	1000	191
2	Same as Case 1			1100	170
3	Same as Case 1			1200	153
4	Same as Case 1			1300	140
5	R = 0.3; d = 0.03581; D = 0.1	8	2.85	1200	165
6	Same as Case 5 but D = 0.15	Same as Case 5			159
7	Same as Case 5 but D = 0.20	Same as Case 5			152
8	Same as Case 5 but D = 0.25	Same as Case 5			146
9	R = 0.3; d = 0.02865; D = 0.2	8	1.8	1200	154
10	Same as Case 5 but d = 0.04300	8	4.1	1200	146
11	Same as Case 5 but d = 0.04300	8	7.3	1200	148
12	Same as Case 5 but d = 0.04300	8	11	1200	155

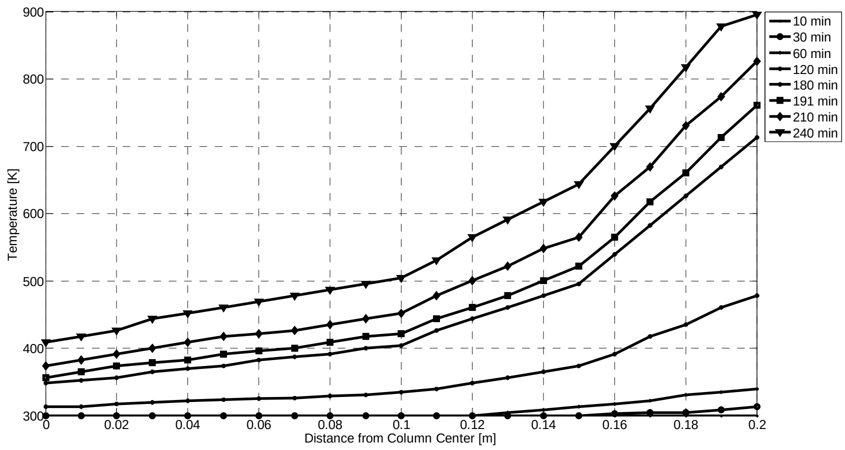

Figure 6: Temperature Distribution for Case 1 along Path 1

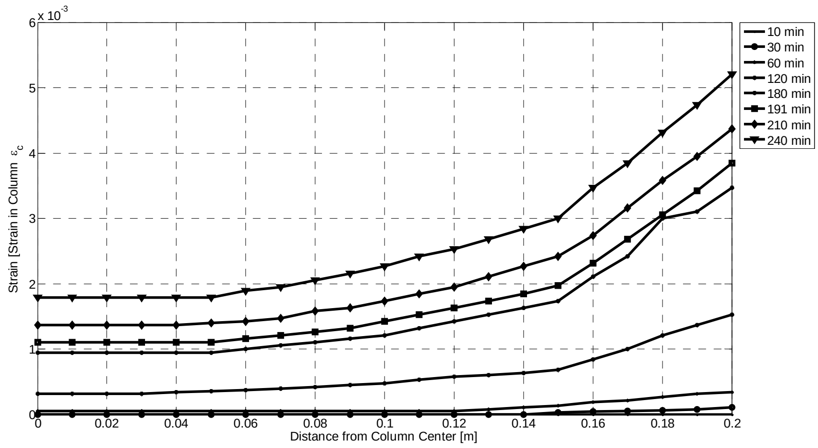

Figure 7: Variation of Strain in Column for Case 1 along Path 1

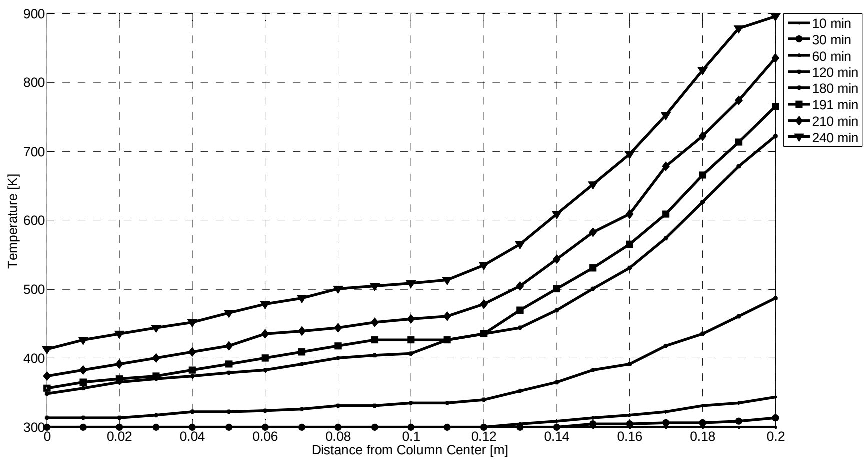

Figure 8: Temperature Distribution for Case 1 along Path 2

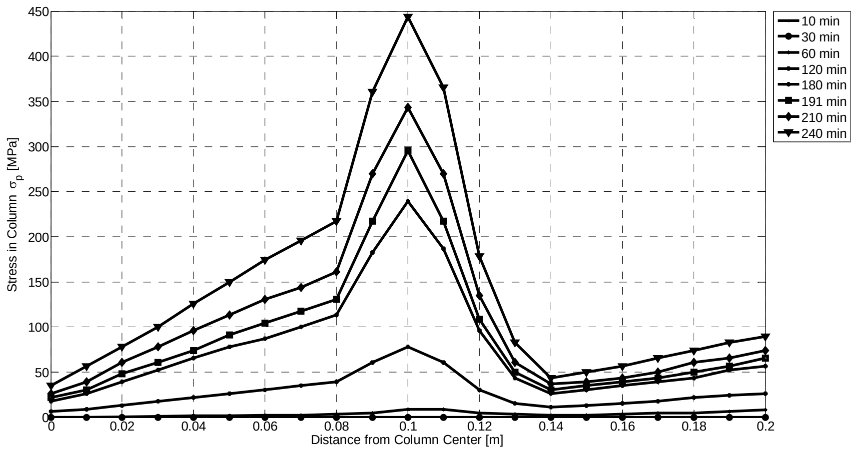

Figure 9: Variation of Stress in Column for Case 1 along Path 2

Figure 8 shows the temperature distribution for case 1 along path 2, which passes through the reinforcing steel bar. The temperature over path 2 increases in the same manner as over path 1 but with distinct difference, that it has a very low temperature gradient, at the location of the steel bar. This is due to the fact that the specific heat of steel is less than it is for concrete. As a result, for the same rate of heat flowing inside the column, the steel will be heated faster than the concrete.

Figure 9 shows the stress variation for case 1 along path 2. As expected, due to the temperature gradient along the column, there is an increase in both stress and strain due to the different rates of expansion. Furthermore, it can be observed concrete elements in the region surrounding the steel bar are more stressed than those elements beyond the steel bar due to the high expansion of the steel relative to the concrete. This variation is primarily due to the fact that steel has a higher coefficient of thermal expansion than that of concrete causing it to expand at a much larger rate and creating a stress concentration in the area between the steel beams.

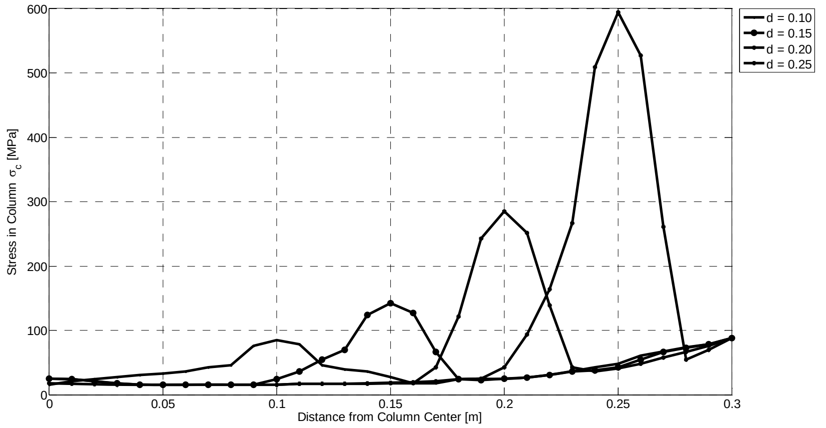

Figure 10: Variation of Stress in Column with Varying Rebar Array Radius (t = 180min)

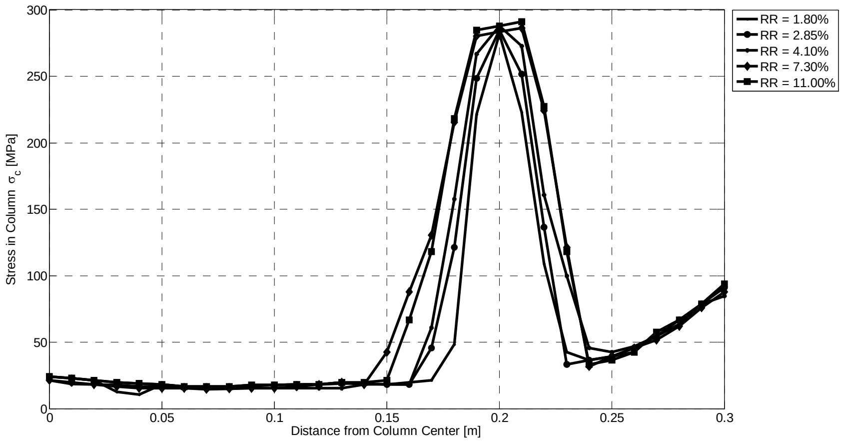

Figure 11: Variation of Stress in Column with Different Reinforcing Ratio

Figure 10 shows the effect of the location of the reinforcing steel bars array on the generated stress. It can be observed that the closer the rebar array to the column's center, the worse the column's resistance to fire. This fact is due to the reason that the rebar array is set in a region closer to the column surface, and therefore it will be more heated, causing the adjacent concrete to be more stressed due to the expansion of the steel bars. Thus, reducing the radius of the rebar array would lead to an increase in the failure time (life time) of the column. On the other hand, it can be noticed that, the closer the array of steel bars is to the column center, the closer the bars are to each other thus any change in temperature of the core of the column will lead to an large increase stress concentration which would result in complete failure of the column.

For cases 9 through 12, the effect of the reinforcement ratio (RR) on the column's failure time, $t_{c,f}$ was investigated. Figure 11 shows the stress distribution over path 2 for different RR, indicating a jump in the stress distribution over path 2 at the location of the steel bar, which agrees with previous cases. As it appears in Figure 12, the column-failure time will decrease from $t_{c,f}$ =154 minutes to approximately $t_{c,f}$ =146 minutes when the Reinforcing Ratio (RR) increases from 1.8 to approximately 5. However, the column's failure time will increase to $t_{c,f}$= 155 min., when RR increases RR=11.

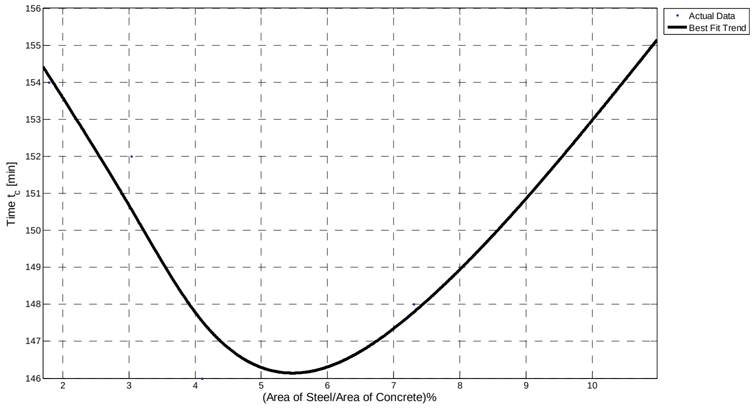

Figure 12: Effect of Reinforcing Ratio on the Time of Failure

The effect of the RR on the failure time is shown in Figure **1**. With a lower RR, the reduction of the steel's mass allows it to heat at a faster rate, causing the adjacent concrete and causes it to be stressed at a higher rate, lowering the column's failure time. Increasing the reinforcement ratio will result in an increasing of the rebar's diameter.

A larger bar diameter will allow the rebar to be in contact with more concrete, thus affecting the overall stress in the concrete. This results in a decrease in the stress in the concrete as well as an increase in the column's time to failure. The expansion of the bar will be less as it requires more heat to increase the temperature of such a larger mass. This process lowers the generated stress in the column resulting in an increased time to failure for the column.

5. Conclusions

Columns constitute the fundamental support of a building, and their failure could lead to the collapse of the entire structure. Furthermore, in time sensitive cases, such as a burning building, the prevention or delay of columns' failure can lead to an increase in time allowed for the evacuation of occupants, recovery of property, and access to the fire. Thus, retarding the failure of load bearing supports in a burning building will delay the collapse of this building and ultimately lead to an increase in the number of lives saved.

Results generated in this work showed a range of significant phenomena regarding the time to failure for concrete columns when exposed to high temperature fire. Furthermore, careful attention must be paid to the placement of the reinforcing beams during the design of the column. The closer to the center the beams, the better resistance to fire but in retrospect they exhibit a lower yield strength due to the stress concentration in the concrete between the beams. However, the reinforcing ratio in the range 4.5% ≤ RR ≤ 6.8% should be avoided to achieve a minimum column failure time of $t_{c,f}$ = 147 min. In addition, designing the concrete to have a lower reinforcing ratio will result in improved fire resistance. Lastly, treating the surface of the column in order to lower the surface emissivity tends to lower the column's time to failure.

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البحث والتحقيق في انهيار أعمدة الخرسانة المسلحة المعرضة للهب النار

مضر الزغول، هاشم الخالدي، سعد حبالي، معتصم جابر، بسام زيادة*

ملخص

يعرض هذا البحث دراسة الاحتمال الزمني لانهيار الأعمدة الخرسانية المسلحة نتيجة تعرضها لارتفاع في درجة الحرارة. إن مثل هذا الارتفاع في درجة الحرارة يشكل مصدر قلق كبير، لأنه يعمل على تغيير الخصائص الميكانيكية لمادة الفولاذ الصلب المستخدم في الخرسانة المسلحة مما يؤدي إلى تدهوره بشكل كبير. تم في هذه الدراسة أخذ أعمدة مختلفة بعين الاعتبار من أجل تحليلها ودراستها مستخدمين مجموعة ثابتة من المعايير ذات العلاقة، وهذه المعايير تشمل ما يلي: المقطع العرضي للعامود، ترتيب قضبان الفولاذ الصلب، قطر قضبان الفولاذ الصلب، بعد المسافة بين مركز القضبان الفولاذية من مركز العامود، عدد هذه القضبان المستخدمة في العامود الواحد، نسبة التسليح، انبعاثية الاشعاع السطحية، ودرجة حرارة اللهب. لقد تم استخدم الكمبيوتر لتحليل هذه المسألة باستخدام طريقة العنصر المحدود ومن ثم حساب توزيع درجة الحرارة على مساحة المقطع العرضي للعامود مع مرور الوقت أثناء تعرضه المباشر للهب. إن ارتفاع درجة الحرارة في العامود يؤدي إلى انخفاض قدرته على رفع الحمولة الخارجية التي قد يتعرض لها، مما يؤدى إلى انهيار المبنى المحمول عد انفعال حرج مقداره ε=0.3% في العديد من الحالات التي تم اشباع البحث والتحقيق فيها، حيث تبين أيضا أن الوقت اللازم لانهيار هذه الأعمدة الخرسانية المسلحة يتراوح ما بين 140 إلى 191 دقيقة.

الكلمات الدالة: استطاعة أعمدة الحمولة، انتقال الحرارة بالحمل والتوصيل والإشعاع، طريقة العنصر المحدود، حرائق المباني، مقاومة الحريق، تسليح أعمدة الخرسانة.

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