The Spiral-Channel Viscous Micropump

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ABSTRACT

In this work, the effect of geometrical design parameters: channel aspect ratio, mean radius to channel width ratio, and spiral curvature ratio on the flow performance of a spiral channel viscous micropump have been reviewed and investigated analytically and numerically. The combined effect of studied design parameters on the flow performance of the spiral pump was also expressed in an approximate model through geometrical drag and pressure shape factors. The analytical estimations were compared with the numerical solution and good agreement was obtained. Numerical results show that the flow rate varies linearly with both the pressure difference and boundary velocity, which supports the validity of the linear lubrication model for this problem for the full range of studied parameters. The obtained extended approximate model depicts complete representation for the influence of channel width, height, spiral curvature, spiral length, and mean radius on the flow performance of a spiral channel viscous micropump.

Keywords: Micropumps, Viscous Flow, Microfluidics.

1. INTRODUCTION

Research on viscous micropumps was firstly introduced in 1996 (Sen et al., 1996), and their development has been governed by the physics of microscale flow, which is characterized by large surface areas to volume ratio, and demonstating viscous forces over inertia. Number of experimental (Kilani et al., 2003, Al-Halhouli et al., 2008, Blanchard et al., 2005), theoretical and numerical studies (Kilani et al., 2006, Al-Halhouli et al., 2007a, Al-Halhouli et al., 2007b, Blanchard et al., 2006) on the viscous micropumps with different configurations has been followed. Eccentric, single and double disk, and spiral channel viscous micropumps are the most viscous micropumps studied thoroughly during last decade.

Investigation of viscous drag micropumps has been motivated by the ability to generate a significant pressure heads in the viscosity dominated micro-channels by the simple rotation of a rigid element contiguous to the flow field. These micropumps are attractive because they are easy to fabricate, capable of handling a wide variety of fluids and can operate with no valves allowing them to handle particle-laden fluids.

Pumping in a spiral channel viscous micropump depends on dragging the fluid along a spiral channel either by rotating the spiral channel disk against a stationary flat disk (moving condition) or rotating the flat disk against the spiral channel (stationary condition). As shown in Fig. (1), the moving micropump condition is obtained by spinning a spiral channel disk in close proximity below a stationary flat disk where the pump chamber was formed. The fluid is dragged from the inlet port located at the center of the stationary disk and pumped outward from the outlet port located at the pump housing. The flow is generated due to a net tangential viscous stress on the boundaries which is built and produces a positive pressure gradient in the direction of flow.

This work reviewed the analytical results introduced recently, develops numerical solution for the effect of spiral curvature ratio, and present an approximate extended flow model estimates for the combined effect of the channel aspect ratio, mean radius to width ratio, and spiral curvature ratio on the flow performance of the spiral channel micropump.

2. SPIRAL CHANNEL MICROPUMP GEOMETRY

The spiral channel viscous pump is comprised of a stationary flat cover and a spinning spiral channel disk...
that forms the pump chamber, with a fluid inlet and outlet ports located at either end of the spiral channel. Figure (1) shows a scheme of the spiral channel pump components. The stationary flat cover is brought to close proximity with the upper walls of the spiral channel creating a small gap with the objective of minimizing leakage from the pump chamber. The height of the pump chamber is the distance between the disk surface and the bottom of the spiral channel, which constitutes the flow passage height of the pump. The idea of micropumping using a rotating disk to drag viscous fluids through spiral protrusion has been recently introduced, (Kilani et al., 2003). The spiral channel centerline of Fig. (1), can be described by the linear “Archemidean” spiral written in polar coordinate as:

$$r = k \theta + r_o , 0 \leq \theta \leq \Delta \theta$$

(1)

Where $r_o$ is the starting radius (value of $r$ at $\theta=0$), $\Delta \theta$ is the angular span, and $k$ is the polar slope of the curve (change in $r$ per $\theta$). The spiral length of the segment is estimated by integrating the differential length over the angular span.

Figure 1. An illustration of the spiral pump (Al-Halhouli et al., 2006).

For the purpose of obtaining an extended flow model estimates for the effect of channel height, width, mean radius, spiral curvature, and spiral length, analytical and numerical investigations have been carried out. In the analytical investigations, models were derived to estimate for the effect of channel aspect ratio and mean radius to width ratio on the flow performance, while the effect of spiral curvature was studied numerically (Al-Halhouli et al., 2007b). Drag and pressure shape factors are generated to express the effect of the pressure difference and boundary velocity on the flow performance at various design parameters.

3. EFFECT OF GEOMETRICAL PARAMETERS

3.1 Effect of Aspect Ratio

The effect of aspect ratio ($w/h$) on the flow performance of a spiral channel viscous micropump has been investigated analytically and compared with numerical simulations (Kilani et al., 2006). An approximate analytical solution has been derived by neglecting the curvature of the spiral and treats it as a straight channel as shown in Fig. (2). This approximation is valid within the unfolding approximation conditions for flows in curved geometries, (Dean, 1927). The obtained results were compared with the numerical simulations at low spiral curvatures ($k/r_o << 1$, $\phi \rightarrow 0.0$). In the case of the viscosity dominated flow in the microscale channels considered in the present analysis (viscous micropump), the viscosity forces are considerably larger than the inertial forces. The inertial and body terms can therefore be completely neglected, and the Navier-Stokes equations can be solved for both stationary and moving wall conditions. The no-slip velocity boundary condition at wall has been also considered through the derivations, where it was found to remain an excellent approximation for liquid flows at scales above tens of nanometers (Stone et al., 2004).

Analytical solution for the volumetric flow rate was found and was expressed as (Kilani et al., 2006)

$$q_A = \frac{U \Delta h}{2} F_{DA} - \frac{h^3 \Delta p}{12 \mu l} F_{PA}$$

(2)

or in dimensionless form
The coefficients $D_A$ and $P_A$ was found to depend on the aspect ratio $w/h$, and can be defined for the stationary and moving wall conditions by (Kilani et al., 2006)

$$A \frac{D_A}{P_A} = \begin{cases} \sum_{n=1}^{\infty} \frac{(-1)^n-1}{n^3} \cosh(n\pi h/w) \sinh(n\pi h/w) & \text{stationary condition} \\ \sum_{n=1}^{\infty} \frac{(-1)^n-1}{n^3} \cosh(n\pi h/w) \sinh(n\pi h/w) & \text{moving wall condition} \end{cases}$$

$$F_{DA,S} = \frac{4w}{h} \sum_{n=1}^{\infty} \frac{(-1)^n-1}{n^3} \cosh(n\pi h/w) - 1$$

$$F_{DA,m} = \frac{4w}{h} \sum_{n=1}^{\infty} \frac{(-1)^n-1}{n^3} \cosh(n\pi h/w) - 1$$

To study the effect of mean radius to channel width ratio $(R_m/w)$ on the flow performance of a spiral channel viscous micropump, the flow is assumed to be dragged in a circular channel with the origin of the polar axis system assigned to the center of the spiral coordinate system with a mean radius of $R_m$ as shown in Fig. (3).
The tangential momentum equation was written in polar coordinates and solved for low Reynolds number flows, and at high aspect ratios \((\frac{w}{h} \rightarrow \infty)\) as

\[
\frac{q_R}{U_ch} = \frac{F_{DR}}{2} \frac{h^3}{12 \mu l} F_{PR}
\]

or in dimensionless form

\[
q_R^* = \left[ 0.5 F_{DR} - \frac{1}{2} F_{PR} R e E u \right]
\]

where \(F_{DR}\) and \(F_{PR}\) are the drag and pressure mean radius to channel width ratio shape factors and are defined as (Al-Halhouli et al., 2006)

\[
F_{DR} = 1.0
\]

and

\[
F_{PR} = \frac{R_m}{w} \ln \left( \frac{2 \left( \frac{R_m}{w} \right) - 1}{\left( \frac{R_m}{w} \right) + 1} \right)
\]

### 3.3 Effect of Spiral Curvature

To investigate the effect of large values of spiral curvature...
The spiral channel viscous micropump will be introduced numerically. Three dimensional CFD numerical simulations have been carried out using finite volume method. Different discretization schemes were analyzed to determine the appropriate grid for simulating the flow field. The structured grid scheme with hexahedral elements was identified to be the most suitable for the present study, as shown in Fig. (5). The process of meshing began by dividing the height edges to be two times the number of their units, while the width edges were divided into divisions equal to the number of width units with 1.206 double sided ratios. The spiral edge of the moving wall was then divided into units equal to the angular span angle, and the volume then meshed with the hexahedral elements. These units were scaled later such that 1 unit length scaled to $1 \mu m$.

Grid independent solution was assured by observing the outlet mass fluxes and the velocity components through the micropump channel. The obtained numerical models were exported to the CFD program, where flow analysis was performed and results were reported.

Laminar, steady, viscous flow model was used for this analysis. Because of its viscous properties SAE10W30 motor oil with $825 \text{ kg/m}^3$ density and $0.0901 \text{ kg/m.s}$ viscosity was defined as the interior fluid. In the boundary conditions menu the values for the outlet pressure and moving walls were assigned for each case. The Simple-C algorithm was used for the pressure velocity coupling; a second order upwind scheme was used for the momentum equations while a second order pressure interpolation scheme was used for pressure. Numerical results for the mass fluxes and boundary conditions were reported and plotted for different spiral curvatures and at different aspect ratios. From theses figures the spiral curvature drag and pressure shape factors were built and plotted as shown in the results and discussion section.

4. EXTENDED FLOW MODEL

The previous analysis, treat the effect of channel aspect ratio, spiral curvature and mean radius to channel width ratio geometrical parameters independent of each others. Since linear behavior for the effect of each parameter on the volumetric flow rate was obtained, the superposition principle was used to express the combined effect of these parameters on the flow performance and an extended flow model can be proposed to estimate for this effect by multiplying the drag shape factors in the Couette flow term by each others and the same for the pressure shape factors in the Poiseuille flow term.

\[
q^* = \frac{1}{2} F_{DA} F_{DR} F_{DK} - \frac{1}{12} Re Eu F_{PA} F_{PR} F_{PK}
\]

The validity of the extended model was verified numerically.

5. RESULTS AND DISCUSSION

To investigate the effect of spiral curvature design parameter on the pump performance, a number of 3D finite volume numerical models were built at different spiral curvatures ratios. This numerical study is motivated by the good agreement of the numerical and analytical results obtained for the spiral channel viscous micropump (Kilani et al., 2006, Al-Halhouli et al., 2006), and the complexity of obtaining 3D analytical solution. Numerical flow rates in dimensionless forms for both the stationary and moving conditions at $\theta = 0$, for various values of $k/r_o$ are shown in Fig. (6a, b). The linear trend is repeated for all values $\theta$, indicating that the flow rate decreases linearly with increasing the pressure difference. This shows that the linear lubrication model employed in (Kilani et al., 2006) for the case of low spiral curvature is still valid for high as well as low spiral curvatures. A deviation of the linear lines from the low curvature results is noticed at $k/r_o > 1.5$. The lines tend to change their slope and intersection points. This change can be shown by plotting the shape factors against the spiral curvature ratio. Increasing the spiral angle increases the normal velocity component value and produces a cavity-like flow.
Plots for the spiral curvature pressure and drag shape factors as a function of spiral curvature ratio can be generated from Fig. (6a, b) using the intercept point for the drag shape factor and the slope for the pressure shape factor at the same channel aspect ratio. Results are shown in Fig. (7) and Fig. (8). Fig. (7) shows that the effect of spiral curvature on the Couette flow term is in between 1 and 0.8, and that it is constant about 1.0 within the range \(0 < k / r_o < 1.5\), and 0.8 at \(k / r_o \geq 5\).

The pressure shape factors are obtained from the slope of the lines built in Fig. (6a, b) and found to be in between 1 and 0.6, and it becomes nearly constant at \(k / r_o \geq 7\).

To verify the validity of the extended flow model presented in Eq. (13). Numerical simulations have been performed and plotted for the non-dimensional volumetric flow rates at different pressure and rotational speed values and compared with the analytical solution. Results for different aspect ratios, mean radius to width ratios and spiral curvature ratios are shown in Fig. (9a, b). Numerical and analytical results are in very good agreement and support the validity of extending Eq. (13) to include the combined effect of the geometrical design parameters.
Figure 7. Effect of spiral curvature ratios on the drag shape factor.

Figure 8. Effect of spiral curvature ratios on the pressure shape factor.
6. CONCLUSIONS

The paper presented analytical and numerical investigations of the effect of geometrical design parameters: channel aspect ratio, mean radius to width ratio, and spiral curvature ratio on the flow performance of a spiral channel viscous pump. An extended flow model which predicts the combined effect of design parameters has also been proposed, and found to be in good agreement with the generated numerical simulations. The maximum error in flow rate in this model has been found to be less than 15%. The flow rate was found to vary linearly with both the pressure difference and boundary velocity for the full range of studied parameters, which supports the validity of the linear lubrication model for this problem.
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Nomenclature

\( Eu \) Euler number
\( F_{At} \) aspect ratio drag shape factor
\( F_{Ar} \) radius drag shape factor
\( F_{Ap} \) aspect ratio pressure shape factor
\( F_{Pr} \) radius pressure shape factor
\( h \) height
\( l \) channel length
\( q \) volume flow rate per unit width
\( P \) pressure
\( r \) radius of curvature
\( R_m \) mean channel radius
\( \text{Re} \) reduced Reynolds number
\( U \) channel velocity
\( \theta \) angular span
\( w \) width

Greek Symbols

\( \Delta \) difference operator
\( \mu \) dynamic viscosity

Subscripts

o initial
A aspect ratio
R radius ratio
ch channel
s stationary wall condition
m moving wall condition

Superscript

\( (\quad)^* \) non-dimensional variable
\( (\quad) \) average

REFERENCES


البحث يتناول تأثير ومعادل أداء جرائد الألومنتية المضغحة في التصميم بعيد الألومن cadenaة، ودور دراسة الأدة على الإجابة السريعة، وفحص النموذج على أساس التغييرات في هذه الألومنة، حيث ت등ع الزمنية للحوادث، والجراين الجاذبة ثم بعد ذلك التحليقة، مما يؤثر على حجم الألومن. ونتيجةً، فإن النتائج المقارنة تبدى وجدت حيث كانت الأعداد الفعلية تحتل أدائها في الألومن والجرائد العامة، مما يؤثر على حادة الألومنة.