Stability of Wind-Turbine Generators

H.M.A. Hamdan*

ABSTRACT
The dynamic stability of a wind turbine-synchronous/induction generator system is considered. Participation factors are used to quantify the contribution of each state variable in the system modes thus the nature of the problem is well understood and defined. The validity of the model used is assessed and the differences from steam powered generators are identified.

Keywords: Stability, Wind, Generators, Participation Factors.

1. INTRODUCTION
Alternative energy sources are attracting lot of interest in recent years (Shelbe, 2009). They are distinguished by higher investment costs, lower running costs, less pollution and greenhouse effects. Wind turbine generators (WTGs) are prominent among these sources. Whether interconnected to a power system or used to supply an isolated load (e.g. a water pump or a reverse osmosis plant with a desalination unit), they can contribute economically and with environmentally acceptable standards to meeting the ever increasing electrical power demand. However, their use is justified only at sites where wind speeds are relatively consistent and high enough to efficiently drive a synchronous/induction generator. Although the contribution of WTGs to the total power demand up to now is modest, yet there is a need to investigate their operational problems under all operating conditions. These problems include modeling, unsteady input torques, methods for aggregating dispersed WTGs, steady state stability, transient stability, and optimization and control (Ortiga- Vasquez, 2007 and Lalor, 2005). Eigenvalues and eigenvectors analysis (Abdel- Majid, 1987 and Hinrichsen, 1982) from which the mode shapes are concluded has been used to analyse such problems but it is well known (Ariaga, 1982) that the participation factors provide a more consistent technique for analyzing power system problems.

This paper addresses the problem of steady state stability of a WTG connected to a large power system through a transmission link. The aim is to identify the characteristics not shared with the previously investigated steam powered synchronous generators (Hamdan, 1989 and Hamdan, 1994). Section 2 outlines the model used while section 3 provides a brief background on the concept of participation factors. Section 4 discusses the relationship between state variables and participation factors under specific operating conditions when the rotor is represented by one lumped mass. The effect of the distributed mass model is investigated in section 5 and differences from lumped mass model and steam powered generators are presented. Section 6 discusses how the system performance is affected by the closure of the exciter feedback loop. The system stability for a wind propeller driven induction generator is assessed in section 7 and differences in the performance from the case of synchronous generators are outlined.

2. SYSTEM MODEL
The system considered comprises a synchronous/induction generator connected to an infinite bus through a transmission link and driven by a wind turbine propeller. Pitch angle control (Muljadi, 2001) is used to regulate the speed of the synchronous generator, which is not needed for the induction generator. The synchronous generator is represented in terms of Park’s equations in a rotor locked frame of reference while the induction generator is described by the fifth order model (Krause, 1987). The wind turbine is the two bladed propeller rotating on a horizontal axis and connected to the turbine via a gearbox similar to the unit built for NASA (Abdel- Majid, 1987). Combining the equations for different components,
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linearising and arranging in the form:

\[
x = Ax + Bu
\]  

(2.1)

Where A and B are matrices of appropriate dimensions and for the system including the synchronous generator

\[
x = [ \Delta i_d, \Delta i_q, \Delta i_{fd}, \Delta i_{kd}, \Delta i_{fd}, \Delta \omega, \Delta \delta, \Delta \theta, \Delta z ]
\]  

(2.2)

\[
u = [\Delta v, \Delta E_{fd}]
\]  

(2.3)

While for the system including the induction generator

\[
x = [ \Delta \Psi_{qs}, \Delta \Psi_{ds}, \Delta \Psi_{qr}, \Delta \Psi_{dr}, \Delta \omega_r ]
\]  

(2.4)

\[
u = [\Delta v_{qs}, \Delta v_{ds}, \Delta v_{qr}, \Delta v_{dr}, v]
\]  

(2.5)

Where

- \( \Delta \) : denotes small perturbation around a mean value.
- \( i_d, i_q \) : direct and quadrature axes currents.
- \( i_{fd}, i_{kd}, i_{dq} \) : direct and quadrature damper winding currents.
- \( \omega \) : instantaneous radian frequency. (lumped mass model)
- \( \omega_1, \omega_2 \) : instantaneous radian frequencies of the wind propeller and generator rotor mass respectively. (distributed mass model)
- \( \delta \) : rotor angle. (lumped mass model)
- \( \delta_1, \delta_2 \) : wind propeller and generator mass angles respectively. (Distributed mass model)
- \( D_{11}, D_{22} \) : self damping coefficients of the wind propeller and generator mass respectively.
- \( D_{12}, K_{12} \) : damping and spring coefficients between the wind propeller and the generator mass.
- \( \bullet \) : denotes differentiation.
- \( \psi_{qs}, \psi_{ds} \) : quadrature and direct axes stator fluxes of I.M.
- \( \psi_{qr}, \psi_{dr} \) : quadrature and direct axes rotor fluxes of I.M.
- \( V_{qs}, V_{ds} \) : quadrature and direct axes stator voltages.
- \( V_{qr}, V_{dr} \) : quadrature and direct axes rotor voltages.
- \( \theta \) : blade angle
- \( v \) : wind speed
- \( K_{ex} \) : Exciter gain
- \( T_{ex} \) : Exciter time constant

\[
z = \frac{d \theta}{dt}
\]

subscripts 1,2: denotes terms referring to the generator and turbine masses respectively

3. SELECTIVE MODAL ANALYSIS

The selective modal analysis technique to be used in this paper has been investigated elsewhere (Ariaga, 1982 and Hamdan, 1989). However, an outline of the analysis is briefly discussed in this section.

Consider an undriven control system described by a set of linear differential equations expressed in the form:

\[
x = Ax
\]  

(3.1)

Assume that the real \( n \times n \) matrix A has n distinct eigenvalues \( \lambda_1, \lambda_2, \ldots, \lambda_n \). Let the corresponding set of right eigenvectors be \( (z_1, z_2, \ldots, z_n) \). Let the set of eigenvectors \( (v_1, v_2, \ldots, v_n) \) be the eigenvectors of \( A^T \) or the left eigenvectors of A. Let the left eigenvectors be chosen such that:

\[
v_i^T z_i = \delta_{ij}
\]  

(3.2)

where \( \delta \) is the Kronecker delta function.

It was shown (Ariaga, 1982 and Hamdan, 1989) that if \( k_{ki} = z_{ki} v_{ki} \) can be considered as the participation of the \( K^{th} \) state when only the \( i^{th} \) mode is excited and therefore, it is a measure of coupling between a mode and a state variable.

4. STABILITY OF THE WIND TURBINE SYNCHRONOUS GENERATOR SYSTEM (LUMPED ROTOR MASS)

The system stability is first studied with the field of the synchronous generator unregulated. The eigenvalues of the system at the loading condition \( p = 0.8, Q = -0.2 \) p.u. are given in table (1).

<table>
<thead>
<tr>
<th>Eigenvalues of the WTG at P = .8 Q = -.2 p.u</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>-.379</td>
<td></td>
</tr>
<tr>
<td>-.673 ± j 3.887</td>
<td></td>
</tr>
<tr>
<td>-.773 ± j 6.626</td>
<td></td>
</tr>
<tr>
<td>-16.398</td>
<td></td>
</tr>
<tr>
<td>-20.857</td>
<td></td>
</tr>
<tr>
<td>-47.03 ± j 375.628</td>
<td></td>
</tr>
</tbody>
</table>

All the real parts of the eigenvalues are negative.
indicating system stability. The association between the modes characterized by the given eigenvalues and the state variables can be fully investigated by determining the participation factors as shown in Table (2). In this table, the participation factors corresponding to one eigenvalue of each conjugate pair is only given, however, the participation factors of the other conjugate eigenvalue are the conjugates of the given participation factors.

The first real eigenvalue characterizes an exponentially decaying mode with the field current (3rd state variable) having the highest contribution in the mode. The second complex eigenvalue characterizes a damped oscillatory mode (referred to as the first mechanical oscillatory mode) with relatively strong coupling to the direct axis variables and very strong coupling to $\Delta \omega$ and $\Delta \delta$ of the rotor (and hence the mechanical mode reference). This mode resembles the oscillation of the whole rotor mass against the electrical system. The third eigenvalue characterizes an oscillatory mode (referred to as the second mechanical oscillatory mode) and is strongly coupled to $\Delta \theta$ and $\Delta z$ state variables. The fourth and the fifth eigenvalues are strongly coupled to the quadrature and direct axis windings respectively. The sixth eigenvalue characterizes a well damped electrical oscillatory mode and is mostly influenced by the parameters of the direct and quadrature axes stator windings.

5. TORSIONAL Modes of oscillation

In the above study of the open loop system stability, the whole rotor system is considered as one lumped mass oscillating against the electrical system. To study the torsional modes of oscillation of the different rotor masses, it is necessary to consider the distributed mass model of the rotor system. Two masses are usually considered, the first is the wind propeller and the gearbox coupled together, and the second mass represents the generator rotor mass. The equations of motion of the two masses are written in the form:

$$M_1 \dot{\Delta \omega}_1 = (\Delta T_m - \Delta T_e) + K_{12} \left( \Delta \delta_2 - \Delta \delta_1 \right)$$  \hspace{1cm} (5.1)

$$\Delta \delta_1 = \omega_0 \Delta \omega_1$$  \hspace{1cm} (5.2)

$$M_2 \Delta \omega_2 = (\Delta T_w - \Delta T_m) + K_{12} \left( \Delta \delta_1 - \Delta \delta_2 \right) + D_{12} (\Delta \omega_1 - \Delta \omega_2) - D_{22} \Delta \omega_2$$  \hspace{1cm} (5.3)

$$\Delta \delta_2 = \omega_0 \Delta \omega_2$$  \hspace{1cm} (5.4)

In this case the new state vector is given by:

$$x = \begin{bmatrix} \Delta i_d, \Delta i_q, \Delta i_{fd}, \Delta i_{iq}, \Delta \omega_1, \Delta \delta_1, \Delta \theta, \Delta Z, \Delta \omega_2, \Delta \delta_2 \end{bmatrix}$$

The system matrix A has a dimension of $(11 \times 11)$. The new eigenvalues will have seven eigenvalues similar to Table (1) with the conjugate eigenvalue $-0.673 + j\, 3.887$ vanishing and the following two conjugate eigenvalues emerging:

- $-0.523 \pm j\, 1.98$ (First torsional mode)
- $-4.29 \pm j\, 25.36$ (Second torsional mode)

The participation factors of all state variables in the new modes of oscillations are given in Table (3).

<table>
<thead>
<tr>
<th>EigenValue</th>
<th>$\Delta i_d$</th>
<th>$\Delta i_q$</th>
<th>$\Delta i_{fd}$</th>
<th>$\Delta i_{iq}$</th>
<th>$\Delta \omega_1$</th>
<th>$\Delta \delta_1$</th>
<th>$\Delta \theta$</th>
<th>$\Delta Z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.379</td>
<td>0.0686</td>
<td>0.0955</td>
<td>1.037</td>
<td>0.033</td>
<td>0.006</td>
<td>-0.057</td>
<td>-0.061</td>
<td>0.0</td>
</tr>
<tr>
<td>-0.673 + j 3.887</td>
<td>-0.3105</td>
<td>-0.053</td>
<td>0.3051</td>
<td>-0.024</td>
<td>-0.0534</td>
<td>0.54</td>
<td>0.5406</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td>-j 0.1655</td>
<td>-j 0.095</td>
<td>+j 0.0188</td>
<td>+j 0.1019</td>
<td>+j 0.0235</td>
<td>+j 0.0395</td>
<td>+j 0.0484</td>
<td>+j 0.0143</td>
</tr>
<tr>
<td>-0.773 + j 6.262</td>
<td>0.0052</td>
<td>0.0032</td>
<td>-0.0003</td>
<td>-0.0033</td>
<td>-0.0017</td>
<td>-0.0025</td>
<td>-0.0036</td>
<td>0.499</td>
</tr>
<tr>
<td></td>
<td>-j 0.0054</td>
<td>+j 0.0003</td>
<td>+j 0.0053</td>
<td>-j 0.001</td>
<td>-j 0.002</td>
<td>+j 0.026</td>
<td>+j 0.01</td>
<td>-j 0.067</td>
</tr>
<tr>
<td>-16.398</td>
<td>0.0153</td>
<td>-2.0942</td>
<td>0.031</td>
<td>-1.3954</td>
<td>3.251</td>
<td>-0.0519</td>
<td>-0.0479</td>
<td>0.0</td>
</tr>
<tr>
<td>-20.857</td>
<td>-1.5267</td>
<td>0.168</td>
<td>-5.111</td>
<td>3.148</td>
<td>-2.391</td>
<td>-0.0174</td>
<td>-0.0198</td>
<td>0.0</td>
</tr>
<tr>
<td>-47.03 + j 375.628</td>
<td>1.5773</td>
<td>1.4508</td>
<td>0.0834</td>
<td>-0.9933</td>
<td>0.9512</td>
<td>0.0 + j0</td>
<td>-0.001</td>
<td>0.0</td>
</tr>
</tbody>
</table>

Table 2. Participation Factors in the System Modes (P = .8, Q = -.2)
The first mode has a low frequency (0.315 Hz) and is strongly coupled to the turbine angular speed and angle. Its coupling to the other state variables is not significant. This means that the turbine mass will oscillate almost independently from the generator rotor mass. This is also different from the nature of oscillations encountered in steam turbine generators where the frequency of oscillation is in the 15 – 40 Hz range and with each mass oscillating with respect the neighboring mass and with almost an equal contribution of its associated state variables (speed and angle) in the mode. The difference is attributed to the use of gearboxes required to match the low turbine speeds to the higher generator speeds (1800/1200 rpm used in 4/6 pole generators). This results in a low mechanical stiffness between generator and turbine (Hinrichsen,1982) when viewed from the generator side and accordingly loose coupling between the two masses.

It is evident from Table (3) that the second mode is strongly coupled to the speed and angle of the much lighter generator rotor mass and the direct axis electrical variables while the coupling to the speed and angle of the heavier turbine mass is very weak. This mode resembles the oscillation of the generator rotor mass almost independently against the electrical system. Compared with the eigenvalue of the first mechanical mode the frequency of oscillation is higher (4.036 Hz vs. .618 Hz) while the damping coefficient is also higher keeping the damping ratio almost constant. The frequency of oscillation is much higher than the lumped mass model due to the much lower mass of the generator rotor now oscillating independently against the electrical system.

6. CLOSED LOOP SYSTEM STABILITY

In this case the exciter loop is closed and the voltage at the generator terminals is regulated. It is assumed that a static exciter is used with the following input-output relationship.

\[
\frac{\Delta E_{fd}}{\Delta (e_{ref} - e_t)} = \frac{K_{ex}}{1 + S T_{ex}}
\]  

(6.1)

It was shown (Hamdan,1994) that keeping the exciter gain relatively low will not cause any system instability. However, at higher gains the second torsional mode in section 5 tends to become unstable while damping of the second mechanical mode is not significantly affected. Standard power system stabilisers (PSS) will be effective in damping this mode of oscillation. Direct methods to design PSS’s are fully investigated (Hamdan,1994).

7. STABILITY OF WIND TURBINE–INDUCTION GENERATOR SYSTEMS

The system in this case consists of a wind turbine driving as induction generator connected to a large power system represented by an infinite bus. Pitch angle control mechanism is no longer required. Based on the assumptions made on section 2 and with the rotor masses represented by one lumped mass, the eigenvalues of the systems are as given in Table (4).

<table>
<thead>
<tr>
<th>Eign value</th>
<th>State Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\Delta d)</td>
<td>(\Delta q)</td>
</tr>
<tr>
<td>-.052</td>
<td>-.0778</td>
</tr>
<tr>
<td>+j 1.981</td>
<td>-j .0192</td>
</tr>
<tr>
<td>-4.29</td>
<td>+j 25.361</td>
</tr>
</tbody>
</table>

Increasing the magnitude of the slip and finding how the eigenvalues change, reveals that the dominant eigenvalue is the first real eigenvalue in Table. (1). It is found that at \(|s| > .052\) the system becomes unstable. The participation factors of the state variables in the system modes are given in Table (5). Examination of this table reveals that the first eigenvalue in Table (4) which characterizes an exponentially decaying mode is strongly coupled to \(\Delta \psi_{qr}\) while its coupling to the other state variables is not significant. The oscillatory mode
characterized by the second conjugate eigenvalue, is strongly coupled to $\Delta \psi_{dr}$ and $\Delta \omega_r$. As for the oscillatory mode characterized by the third conjugate eigenvalue, it is strongly coupled to $\Delta \psi_{qs}$ and $\Delta \psi_{ds}$.

Table 5. Participation Factors for Wind Turbine – Induction Generator System $S = -0.0078$

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>State Variable</th>
<th>$\Delta \psi_{qs}$</th>
<th>$\Delta \psi_{ds}$</th>
<th>$\Delta \psi_{qr}$</th>
<th>$\Delta \psi_{dr}$</th>
<th>$\Delta \omega_1$</th>
<th>$\Delta \omega_2$</th>
<th>$\Delta \delta_2 - \Delta \delta_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-19.676</td>
<td></td>
<td>0.0</td>
<td>-0.0037</td>
<td>1.0158</td>
<td>-0.0134</td>
<td>0.0013</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-8.793</td>
<td></td>
<td>-0.0018</td>
<td>0</td>
<td>-0.0062</td>
<td>0.5086</td>
<td>0.4993</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+j 43.987</td>
<td></td>
<td>+j 0.0039</td>
<td>+j 0.0002</td>
<td>+j 0.0002</td>
<td>+j 0.1256</td>
<td>-j 0.1322</td>
<td></td>
<td></td>
</tr>
<tr>
<td>-26.68</td>
<td></td>
<td>-0.5018</td>
<td>-j 0.0008</td>
<td>-0.0017</td>
<td>0.0001</td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>+j 375.65</td>
<td></td>
<td>+j 0.0008</td>
<td>-j 0.0003</td>
<td>+j 0.0006</td>
<td>+j 0.0006</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The effect of considering the rotor as composed of two masses namely the heavy wind turbine and the much lighter induction generator rotor is also studied. The new state variables vector in this case is given by:

$$[x] = [\Delta \psi_{qs}, \Delta \psi_{ds}, \Delta \psi_{qr}, \Delta \psi_{dr}, \Delta \omega_1, \Delta \omega_2, (\Delta \delta_2 - \Delta \delta_1)]$$ (7.1)

The eigenvalues of the new system are given in Table (6).

Table 7. Participation factors For Wind Turbine – Induction Generator $S = -0.0078$ (distributed mass model)

<table>
<thead>
<tr>
<th>Eigenvalue</th>
<th>State Variable</th>
<th>(One lumped rotor mass)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\Delta \psi_{qs}$</td>
<td>$\Delta \psi_{ds}$</td>
</tr>
<tr>
<td>-0.03 + j 2.90</td>
<td>0.0 + j 0.0</td>
<td>0.0 + j 0.0</td>
</tr>
<tr>
<td>-10.15 + j 10.01</td>
<td>-0.0019 + j 0.0</td>
<td>-0.0002</td>
</tr>
</tbody>
</table>

8. CONCLUSIONS

The participation factors concept has been successfully applied to investigate the problem of the dynamic stability of wind turbine generators. It provides an enlightening insight into how much the system modes are affected by the system state variables and thus defines in a very direct manner the nature of the problem, if it exists, paving the way for the selection of corrective measures to improve the system performance.

It has been shown that the use of a lumped mass model for all the rotor masses will give misleading information on the system stability of a WTG system employing a synchronous generator. A distributed mass model should be used in order to have an accurate insight to the dynamic stability problem. It is shown that the
rotor masses are almost decoupled from each other during these oscillations. Hence no excessive torsional torques are expected during disturbances and these torques are very much lower than those encountered in steam powered synchronous generators. The rotors of the steam powered generators oscillate against the electrical system at a frequency in the 1 – 3 Hz range (Hamdan, 1989) while the frequency of oscillation of the rotors in wind turbine driven generators is greater than 3 Hz. System instability of the regulated system may occur at high leading reactive power deliveries in the same manner encountered in steam powered synchronous generators and PSS can then used to stabilise the system.

As for wind turbine-induction generator systems, the dominant eigenvalue is not significantly affected by the method of modeling of the rotor (lumped/distributed), however, the damping and the frequency of the other mode coupled to $\Delta \psi_r$ and $\Delta \omega_r$ (direct axis rotor flux and rotor speed) is deeply affected by the modeling method. This mode is usually retained in all reduced order models of the induction machines and hence care should be taken when this mode is studied to adopt the correct modeling procedure.

REFERENCES


الهوائية التوربينية موالدة الاستقرارية

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البيئة الاقتصادية الظروف المتسوية الكهربائية للتوليد بكثرة الهوائية التوربينية موالدة تستخدم.

موالد تدير الهوائية التوربينية من تتألف الهوائية التوربينية موالد النظام الثابتة الاستقرارية مسألة البحث هذا يتناول متناوباً ومرت بمعالجة الأشكال المختلفة يقتمل حديثاً الهواء، خط خلال كبير كهرباً بتنظيم نظمه، ويتهم خطوة القياسية بها في تربيبه،

الذاتية القيمة بانتشار واحدة بكتلة الدوار يمثل فيه يتم متناوباً موالد تدير الهوائية التوربينية دراسة تتم،

وعامل بكتلة الدوار يمثل عند الدراسة تستكمل ثم وآن النظام، أنماط في الحالة متغيرات مساهمة في تحديد المشاركة حديثاً الموالد تدش،

الにくい، خاردة النتائج يعطي واحدة بكتلة الدوار يمثل يمكن أن النظام، تقريرية تبدُب الهوائية التقليدية التجارية الهوائية ببدلاً فعلى تنزده,

للمتوسط، إذا، هواد، توافق، بطرق عميقة تشترك الهوائية أنماثاً فأن حديثي، موالي على الدراسة،

الكليلة: المشاركة العاملات، القيم الولد، الرياح، الاستقرارية،.

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