A synchronized multi-stage single product Kanban controlled production line is designed in such a way that yields a feasible operating cost as well as a feasible average of Work In Progress (WIP). The kanban production line is formulated as a queuing network model. Each node in the network is considered as $(M/M/s; GD/∞/∞)$ queue. A new approach for analyzing the production network model is discussed with synchronization mechanism. An operating cost model is then developed to determine the unknown parameters of the system. Numerical examples are used to demonstrate the computations of different system parameters. Avenue for future research is also highlighted.


1. INTRODUCTION

The design of a multi stage Kanban controlled serial production line that produces a single product type is considered as the problem of this work. This Kanban line contains Work In Progress (WIP) parts that are currently being processed. The finished products that have completed processing are stored in the output buffer of the production line. The raw material enters the line after receiving it from the suppliers, and the WIP, as such flows through the line. The process of transporting WIP continues until the finished product departs the last stage and is delivered to the customer in time, the output rate of the last stage is generally dictated by the customer’s demand.

An important executive concern is how to control and synchronize the flow of materials in the line so as to build a consistent integrated shop floor control system that successfully meets the customer demand Just-In-Time (JIT).

2. Literature Review

Several researches have contributed to the development of many approaches that deal with the above mentioned concern, among which are the Kanban control policy that was invented in Toyota in the 1970, and has since been widely used in industry, Monden (1983); Flexible Kanban system discussed by Surendra et al. (1999); The CONWIP control policy as described and analyzed by Gstettner et al. (1996) and Wen et al. (2001), The Base Stock Control policy, Scott et al. (2003); The Generalized Kanban control system introduced by Buzacott (1989); The Generic Kanban systems for dynamic environment modified by T.M. Chang et al. (1994); The extended Kanban control system proposed and discussed by Claudine et al. (2000) among others. A comparison of these control policies as well as other policies can be found in Liberopoulos et al. (2000) and Zhao et al. (2002). Deep exploration of the tradeoffs using analytical models could help differentiate regions where each policy dominates over the other. Ananth et al. (2001) state that queuing models seem to be the most useful tool for such analysis. They modeled Kanban controlled production line as a closed cyclic queuing networks with manufacturing stations and fork/join synchronization stages. Stefan et al., (1998) present a generalized analysis technique for queuing networks with mixed priority strategy and class switching; they show how to transform a queuing network that cannot be solved into a network model that can be solved using a standard analysis technique. A general purpose analytical method for performance evaluation of multistage Kanban
controlled production line is developed by Yves Dallery (1996), the system is modeled as a queuing network, then decomposed into a set of subsystems each being associated with a particular stage and analyzed in isolation using a product form approximation technique.

Akturk et al. (1999) present a literature review and a classification of techniques to determine both the design parameters and kanban sequences for JIT manufacturing systems. Bhaba et al. (1999) constructed a cost function that is developed based on the cost incurred due to the raw material, WIP between stages, and the finished goods. They obtained optimal numbers of raw material orders that minimize the total cost function, which is used to find the optimal number of Kanban.

3. Objective

The principal objective of this work is to present a synchronization mechanism that operates a multi-stage single product Kanban controlled production line in such a manner that keeps a feasible WIP level going in the line. Stage’s production capacity and processing rate, utilization factor of the system, number of processors in the system, and the ordering rate of raw material are considered as the main design parameters.

4. Kanban Discipline

As shown in Figure 1, withdrawal of a product from the finished parts inventory by the demand triggers the system as follows: As a full container is removed from the output inventory of station (i), the production ordering Kanban is detached from it and is posted at the production ordering Kanban post of the same station (i) to signal the need for production of equivalent numbers of units, which are withdrawn. Station (i) can start operation when, (1) it is free, (2) there is a free withdrawal Kanban at the output queue and (3) there is a withdrawal Kanban attached to a full container at the input queue.

As the full container is withdrawn into station (i), the withdrawal Kanban is detached from the container and is posted at the withdrawal Kanban post at the input queue of station (i). Part carrier collects the withdrawal kanbans from their post at station (i) and moves them to station (i-1). If there is a full container at the output queue of station (i-1), the production ordering Kanban is detached from it and posted at the production ordering Kanban post of station (i-1) while the withdrawal Kanban from station (i) is attached to the full container and is moved to the input queue of station (i). The production ordering Kanban at its post now signals station (i-1) for production of equivalent number of parts that are withdrawn. This process is repeated upstream and downstream of the line as the products are processed according to the demand at each stage.

5. Notations

To model the different interactions of the system variables, the following notations are used:

- \( \tau_i \): Lead-time demand or processing time for one container in stations \( i \) (Unit Time).
- \( D_C \): Outside demand rate received from Customer (Units/Unit time).
- \( D_i \): Quantity demanded by station \( i \) (Units/Unit time).
- \( K_{iC} \): Number of the customer withdrawal Kanban.
- \( n_K \): Size of customer withdrawal Kanban (Units).
- \( \mu_i \): Overall processing rate for stage \( i \) (Units /Unit time).
- \( s_i \): Number of processors installed in the stage \( i \).
- \( \mu_{di} \): Processing rates for each processor at stage \( i \) (Units /Unit time).
- \( \rho_i \): Overall Utilization percentages of station \( i \).
- \( \lambda_i \): Arrival rate of WIP into stage \( i \) (Units/Unit time).
- \( WIP_i \): Average unit of WIP exit in station \( i \) at any time (Units).
- \( WIP_{exit} \): Average unit of WIP exit in the flow shop line at any time (Units).
- \( K_i \): Production Kanban number at station \( i \).
- \( WK_i \): Withdrawal Kanban number at station \( i \).
- \( WK_c \): Customer Withdrawal Kanban.
- \( n_i \): Kanban size implemented in station \( i \).
- \( \pi_{pi} \): Probability that all processors in station \( i \) are idle
- \( \ell_i \): Lead-time, or average time product spends in the queue of station \( i \), or flow.
- \( b \): Unit purchasing cost of raw material ($/Unit).
- \( o_i \): Cost of having and transporting one withdrawal Kanban at station \( i \) ($/Unit Kanban).
- \( o_{ci} \): Cost of having and withdrawal Kanban by the customer ($/Customer withdrawal Kanban).
- \( p_i \): Unit processing cost at station \( i \) ($/Unit).
- \( f_i \): Fixed installation cost per one server at station \( i \) ($/Server).
- \( c_i \): Cost of shipping a quantity of one Kanban from station \( i \) to station \( i+1 \) ($/Kanban).
- \( h_i \): Unit holding cost at station \( i \) ($/Unit Time).
- TOC: Total operating cost of the system to produce the desired demand ($).

6. Assumptions

The approach introduced here has been presented
According to the following assumptions:
1. At any time, there will be no demand shortage in the system. The shortage cost of the finished product is zero. Otherwise, customers will wait for a period of time before receiving finished products; therefore, products will not be delivered on time.
2. Since the line considered produces only one type of product, therefore, set-up cost and change over time are neglected.
3. Zero safety lead-time factor, there is no safety stock in the considered line.
4. System stage queues are modeled as (M/M/s: GD/∞/∞).
5. When a job leaves one stage, the probability that it will go to another station is independent of its past history and of the location of any other job.
6. For each station the number of withdrawal kanban is equal to the number of production kanban.
7. Customer demand is constant and deterministic.

7. Queuing Network Model
Consider a system with $i = 1, 2$. $N$ serial processing stages that produce a certain product, as shown in Figure 2. The last stage ($i = N$) in the kanban line receives demand from an external customer. Stage ($i = N-1$) receives demand from Stage ($i = N$) in a rate equal to the processing rate of Stage ($i = N$). Consequently, the demand moves upstream to the first stage ($i = 1$) which is triggered to produce the demand of Stage ($i = 2$). Generally, we can express the demand of stage $i$ as the processing rate of stage $i - 1$. The production rate at stage $i$, and demand rate at station $i + 1$ can be compared with the arrival and service rates of a typical queuing system.

In the queuing system, customers arrive at the arrival rate and wait to be serviced, after which they depart. Similarly, for station $i$ in the kanban system, materials are produced at the production rate of station $i$ and depart at the demand rate for station $i + 1$. The conditions of a pull system require that materials exit stage $i$ only at the rate desired by stage $i + 1$. Hence, demand rate for stage $i + 1$ serves as the rate which departs from $i$ and so the demand rate is equivalent to the service rate in a queuing system, the argument for similarity between arrival and production rates is analogous.

The Units of WIP will pass through a series of queues arranged in a network structure. The flow of WIP is serial. Each node (stage) of the network is modeled as a single unlimited capacity queue and one or more identical parallel processors. All the stages in the network can be modeled as (M/M/s: GD/∞/∞) queues and each station can be analyzed independently using the formulas of (M/M/s: GD/∞/∞) queue models.

Shortage will be avoided if we set the target inventory of any stage equal to the lead-time demand $(\tau_i, D_{i+1})$.

\[ K_i \geq \frac{\tau_i D_{i+1}}{n_i} \]  \hspace{1cm} (2)

And hence, the minimum number of kanban to ensure replenishment before lead-time demand is consumed is:

\[ K_i = \frac{\tau_i D_{i+1}}{n_i} \]  \hspace{1cm} (3)

\[ \tau_i = \frac{n_i}{\mu_i} \]  \hspace{1cm} (4)

Synchronizing the network will start from the last stage, at which the outside demand rate equals to \( k_c n_c = D_c \) \hspace{1cm} (5)

and \( k_in_i = D_i = \lambda_i \) \hspace{1cm} (6)

The last stage must be designed to include a number of processors that satisfy the customer’s demand with an acceptable utilization factor. Therefore, the processing rate for the last stage must equal

\[ \mu_N = D_C \] \hspace{1cm} (7)

\[ \mu_i = D_{i+1} \] \hspace{1cm} (8)

With a utilization percentage of \( \rho_i \) for station \( i \) that include \( s_i \) identical parallel processors, and to keep the line synchronized with the outside customer demand, stage \( i \) must be fed with WIP in a rate (arrival rates) equal to

\[ \lambda_i = \frac{\mu_i}{\rho_i} \] \hspace{1cm} (9)

\[ \mu_i = \frac{s_i}{\mu_i s_i} \] \hspace{1cm} (10)

or

\[ \lambda_i = \frac{s_i}{\mu_i s_i} \rho_i \] \hspace{1cm} (11)

\[ \lambda_i = (D_i) \text{(State Transition Equation)} \] \hspace{1cm} (12)

Based on the above stage, \( (i) \) in the network is modeled as (M/M/s: GD/∞/∞) queue, consequently,
average WIP units exit in the kanban stage (i) at any time, and lead-time can be expressed by equation 14 and equation 16, respectively.

\[
WIP_i = \left[ \frac{(s_i \rho_i)^2 (\sigma_{0i}) (1)}{\left(\frac{s_i}{1-\rho_i}\right)} \cdot \frac{1}{\lambda_i (1-\rho_i)} + \frac{1}{\mu_{si}} \right] \]  
(14)

\[
\pi_{qi} = \frac{1}{\sum_{n=0}^{\infty} \left(\frac{s_i}{n}\right)^n + \left(\frac{s_i}{n}\right)^n} \]  
(15)

\[
\ell_i = \frac{(s_i \rho_i)^2 (\sigma_{0i}) (1)}{\left(\frac{s_i}{1-\rho_i}\right)} \cdot \frac{1}{\lambda_i (1-\rho_i)} + \frac{1}{\mu_{si}} \]  
(16)

The average WIP in the total network is

\[
WIP_{\text{Total}} = \sum_{i=1}^{N} WIP_i = \left[ \sum_{i=1}^{N} \left(\frac{(s_i \rho_i)^2 (\sigma_{0i}) (1)}{\left(\frac{s_i}{1-\rho_i}\right)} \cdot \frac{1}{\lambda_i (1-\rho_i)} + \frac{1}{\mu_{si}} \right) \right] \]  
(17)

8. Operating Cost of the System

A minimum level of WIP yields lower total inventory cost as well as lower unit production cost. Therefore, minimizing WIP inventory level is supposed to be considered as an important strategy when designing, operating, and/or managing any production system. Vijay et al. (1998) estimated total cost of WIP in a kanban controlled production network based on: the shortage cost per kanban per unit time and the unit holding cost. Another approach for estimating cost of WIP in a kanban controlled production is modeled by Bhaba et al. (1999). The expected total cost of WIP in this model includes, the cost of having a kanban at the stage and holding cost only.

For the current developed system, the Total Operating Cost (TOC) consists of four main components: the first one, \(RMOC\), which is the cost elements which constitute raw material purchasing cost \((b \$/purchased unit)\) and cost of having and ordering a withdrawal (supplier) kanban \((c_r \$/supplier Kanban)\) among all stages. The purchased quantity is equal to the demand rate of first stage to be provided by supplier \(D_1\).

\[
RMOC = b (D_1) + \sum_{i=1}^{N} c_r (WK_i) \]  
(18)

It has been noticed that \(D_1 = \lambda_i\), and the customer should pay the cost of his order \(o_i + (WK_i)\).

The second cost component, \(SMC\), which is the total direct and indirect station manufacturing cost occurred in all system stages, this cost depends on: unit processing cost at stage \(i\) and is given by \(p_i (\$/unit)\) and the fixed installing cost of processors in the same stage.

\[
SMC = \sum_{i=1}^{N} \left[ p_i (\mu_i) + f_i (s_i) \right] \]  
(19)

The next cost component, \(TSC\), which is the transportation cost of all demanded quantities to all and shipment cost to the final customer. Transportation cost depends on the cost of having and transporting a quantity of one Kanban from stage \(i\), \(i + 1\), \(c_r\), \(\$/Kanban)\).

\[
TSC = \sum_{i=1}^{N} \left[ c_r (K_i) \right] \]  
(20)

The last cost component is, \(WIPC\), which represents the total holding cost of WIP in the system and the estimation based on the average unit holding cost is:

\[
WIPC = \sum_{i=1}^{N} \left[ h_i (WIP_i) (\ell_i) \right] \]  
(21)

Now, the total operating cost of, \(TOC\) can be written as shown in the following equations:

\[
TOC = RMOC + SMC + TSC + WIPC \]

\[
TOC = b (D_1) + \sum_{i=1}^{N} \left[ c_r (WK_i) \right] + \sum_{i=1}^{N} \left[ p_i (\mu_i) + f_i (s_i) \right] + \sum_{i=1}^{N} \left[ c_r (K_i) \right] + \sum_{i=1}^{N} \left[ h_i (WIP_i) (\ell_i) \right] \]  
(22)

\[
\frac{\partial (TOC)}{\partial (WIP_i)} = \sum_{i=1}^{N} \left[ h_i (\ell_i) \right] \]  
(23)

9. Numerical Results

In this section, we study the behavior of the method proposed in this paper. We tested the method on several scenarios, obtained by the different customers’ demands, the number of kanban implemented in the line, the utilization rate of the manufacturing process at each stage, and the number of servers in each work station assuming a maximum server processing rate for each server. For such analysis, the kanban-controlled production line in Figure 3 is considered. Given \(o_i = [20,
11, 13, 9 $/Withdrawal Kanban], $f_i = [200, 250, 200, 300 $/server], $c_i = [10, 11, 13, 9 $/ Kanban], $h_i = [0.1, 0.2, 0.1, 0.2 $/unit/day], $b = $10/unit. The results are shown as in Table (1). It is clearly shown that the minimum system operating cost is for the first scenario where $s_i = [1,1,1,1$ Server], and the production line will be synchronized when $\lambda_i = [13.12, 14.58, 16.20, 18.00$ units/day] and $\mu_i = [14.58, 16.20, 18.00, 20.00$ units/day] with a total operating cost of $13862. As expected, the total operating cost decreases when the number of servers decreases.

Consider now the Kanban numbers implemented in the line at each. The analysis is done based on the total number of kanban implemented in the line and stage under the above conditions. As is shown in Table (1), the total Kanban number for the first scenario is 6 $(1+2+2+1)$. As it has been noticed, the withdrawal kanban is assumed to be equal to production kanban. The relation between the total Kanban number and the total operation cost is represented in Figure 4. As the expected cost increases, the total kanban number increases, that is because the total inventory cost as well as the total production cost are directly proportional to the number of kanbans.

One of the main features of the proposed synchronizing method is that the processing rate and arrival rate (demand) at each station can be dynamically adapted to the increase or decrease of the outside customer demand. This idea is simulated in Table (2) for different scenarios of customer demand at a utilization rate of 90% and under the same condition related to Table (1).

The synchronization feature of the system is clearly shown in Figure 5, which shows how WIP unit remains constant in the same stage regardless the change of customer demand. Also, WIP remains the same regardless the change of $\lambda_i$ or $\mu_i$. Figure 5 is obtained under the same condition as mentioned before.

It is also worth testing the effect of utilization rate on the system parameters, Figure 6 demonstrates the impact of utilization rate of different system parameters, as the utilization rate increases WIP also increases, this is expected because more need for inventory is required as utilization rate increases to reduce the starvation of servers.

Finally, the tested model sensitivity for changing server’s number implemented in the system, for the analysis we assumed the same number of servers for all stages. Under the condition of $\mu_i > \mu_s$, nothing has been changed except that the total operating cost is increased in the manner shown in Figure 7.

10. Conclusions

The Japanese Kanban technique to achieve the goal of lean production environment has been observed to minimize the Work In Progress (WIP) as well as to minimize inventory cost. The research issue considered here is one of the problems faced today in kanban-controlled production lines. The research has considered a queuing synchronizing mechanism that simulates the interaction of the different system parameters together. The research has proved that customer demand rate can dictate the values of the system parameters, such as the value of the number of kanban needed in the system, system capacity, processors needed at a specific utilization rate, number and sizes for orders of raw material needed from outside supplier. All of these parameters are dependent on one another and they dictate the system operating cost together. These results have been raised through the analytical analysis of numerical example. Results indicated that the presented synchronizing approach is reliable for determining a multi stage kanban line. It is necessary to highlight this point of the research that the best design when $s_i = [1,1,...1]$ under the condition of $\mu_i = \mu_s$. Future research may focus on the issues of multi stage serial assembly lines for multi products.

Acknowledgement

This research has been supported by The Deanship of Academic Research, University of Jordan.
Figure 1: Kanban discipline in a multi stage serial production line.

1. Withdrawal Kanban Post
2. Production Kanban attached to full containers at output
3. Processing center
4. Production Kanban Post
5. Full containers at the output queue of the stage

Figure 2: Queuing network modeling of kanban controlled production line.
Table 1: System results for customer’s demand of 20 units/day under different scenarios, at $\rho = 90\%$ for all stages.

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Figure 3: Kanban controlled line example

Figure 4: Impact of the total number of kanban on total operation cost
### Table 2: System results for different customer’s demand under the conditions as in Table 1 scenario1

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**Figure 5: Design of a Synchronized Stage.**
Figure 6: Impact of utilization rate on system parameters.

Figure 7: Relation between number of server’s $s_i$ and TOC.

REFERENCES


Dallery, Yves. 1996. An Analytical Method for Performance
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