Generalized Performance Analysis of Communication Networks

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ABSTRACT

Communication networks have been considered for performance analysis evaluation when operating under conditions of random failure using computer simulation. The concepts of structural reliability and operational reliability were proposed as the two problem classes for performance analysis. For structural reliability, the network failure probability was adopted as a performance measure. For operational reliability, the performance parameters were defined in terms of Lost Call Traffic for circuit switched networks, and of Average Network Delay for packet switched networks. A generalized algorithm for performance evaluation was developed and tested through network examples to illustrate the applications of these methods and their suitability for network performance analysis.

KEYWORDS: Graph model, Network performance, Random failure, Circuit switched, Packet switched, Structural reliability, Operational reliability, Failure Probability, Lost Call traffic, Average delay, Algorithms.

1. INTRODUCTION

Current Communication networks have become integrated to serve both traditional communication traffic and computer multimedia traffic. The Internet is an example demonstrating the extent of this integration. Depending on the kind of traffic, the mode of transmission could include the employment of circuit switched or packet switched networks.

Performance measures of such networks depend, in part, on the conditions of operation the network is assumed to operate under, and in part on the level of quality in their structural design. For example, the influence of failure whether intended or occurring randomly, will suggest certain parameters as a measure for network performance. For all types of networks, which can be modeled by linear graphs, typical parameters for network structural reliability include two-node or all-node reliability. Other parameters specific to circuit switched networks include Lost Call Traffic (LCT), while for packet switched networks, the Average Time Delay for a packet to be transmitted across the network is typical.

Recognizing that typical networks in common use are large in size, and that closed form analytic solutions for evaluating such parameters are not feasible, computer simulation becomes the common approach for obtaining acceptable solutions. Therefore, in this paper we investigate the various simulation techniques available, and try to develop a generalized algorithm to evaluate the appropriate performance parameters for communication networks under conditions of random failure using computer simulation.

2. Performance Measures

For a communication network, designed, installed, and operating, a common and important question is how reliable this network is? The reliability question here is concerned with the network topology and how well it was designed. The next level of performance would be to evaluate an appropriate reliability measure when the network structure is subject to random failure given that nodes and links are designed with a certain probability of failure. The final level of performance would be to test the network ability to carry traffic subject to varying load levels and routing constraints, and the effect of random failure on such ability to carry traffic. It is convenient

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therefore, to classify reliability as a performance measure into structural reliability and operational reliability.

3. Notations

In our simulation study, a network is modeled by a graph \( G(\mathcal{N}, \mathcal{L}) \), with \( \mathcal{N} \) representing the set of nodes and \( \mathcal{L} \) representing the set of links. When random failure occurs, it affects links or nodes independently. Thus, failure events are statistically independent. Without loss of generality, we assume only links are subject to failure, since node failure can be accounted for by removing the failed node and all links incident at it. The following notations are needed in subsequent development of certain algorithms of interest.

\([a_{ij}]\) denotes the \( N \times N \) connection matrix defining the graph model of the network.

\( a_{ij} \): connection matrix entry equal 1 if node \( i \) is connected to node \( j \), and equal to zero if not connected.

\( p_i \): failure probability of link \( i \).

\( x_i \): link state binary variable: \( x_i = 1 \) if link \( i \) operates, otherwise \( x_i = 0 \).

\( X_j \): network state vector of sample \( j \); \( X_j = \{ x_i: i = 1, 2, \ldots, L \} \).

\( G_j \): graph realization of state \( x_j \).

\( X \): set of all network states \( X_j \).

\( p(x) \): probability that network state \( x \) occurs, and is given by:

\[
p(X_j) = \prod_{i \in \mathcal{L}} \left( p_i + (1-2p_i)x_i \right) , \quad X_j \in X
\]

\( \phi_{s,t}(x_j) \): A binary function equal to 1 if nodes \( s \) and \( t \) are connected in state \( X_j \), otherwise it is equal to 0.

\( \Phi(x_j) \): A binary function equal to 1 if network state \( X_j \) is disconnected, otherwise it is 0.

\( F(s,t) \): probability that nodes \( s \) and \( t \) are not connected in \( G \).

\( R(s,t) = 1 - F(s,t) \): Probability that nodes \( s \) and \( t \) are connected in \( G \).

\( F(G) \): Probability that \( G \) is disconnected.

\( R(G) = 1 - F(G) \): Probability that graph \( g \) is connected.

\( \rho \): Number of components in \( G_j \).

\( g_i \): Component \( i \) in \( G_j \), \( i = 1, 2, \ldots, \rho \).

\( \theta(g_i) \): Number of nodes in component \( g_i \).

\( g(n_i) \): The component containing node \( i \).

\( \hat{r}(l_i), \underline{t}(l_i) \): the terminal nodes of link \( i \).

4. Structural Reliability

Here we assume a given network with a specified topology and certain design parameters associated with links and nodes. For such a network, two approaches are considered in defining and measuring the parameters representing structural reliability. One is deterministic, and the other is probabilistic. In deterministic reliability, we examine the weakest aspect of network topology, usually specified in terms of finding the minimum link cut set, or the minimum node cut set in the network graph model. Thus, reliability is indicated by the failure of no less links or nodes than the minimum cut set before the network is considered as failed.

In probabilistic reliability, link or node failure occur randomly, and as a result, network failure becomes a random event possessing a probability which can be calculated as the performance measure of interest. Since links and nodes are designed with certain failure probabilities, network failure probability is usually expressed in terms of link and/or node failure probabilities.

4.1 Deterministic Reliability

The important issues addressed in this type of reliability are concerned with the structure of the network and its resistance to preplanned forms of destructive forces. As a measure of reliability, cohesion, \( \gamma \) and connectivity, \( \omega \) have been commonly used.

Let \( G(\mathcal{N}, \mathcal{L}) \) define a graph model of any network of \( N \) nodes and \( L \) links. If \( \gamma_{ij} \) is the minimum \((i,j)\) link cut set with respect to nodes \( n_i \) and \( n_j \), then cohesion \( \gamma \) is defined as the minimum \((i,j)\) link cut set for all node pairs of \( G \) such that

\[
\gamma = \min_{i,j} (\gamma_{ij}) \qquad \ldots (1)
\]

Similarly, if \( \omega_{ij} \) is the minimum \((i,j)\) node cut set with respect to nodes \( n_i \) and \( n_j \), then the node connectivity \( \omega \) of \( G \) is given by:

\[
\omega = \min_{i,j} (\omega_{ij}) \qquad \ldots (2)
\]

It follows as, given by the Whitney inequality, that (Wilkov, 1972):

\[
\omega \leq \gamma \leq 2L/N \qquad \ldots (3)
\]
To calculate \( \gamma \), an algorithm due to Ford and Fulkerson, called Augmentation Algorithm (Cravis, 1981) can be used to find the maximum flow between nodes \( n_i \) and \( n_j \) after assigning capacity one to every directed link. This maximum flow is equivalent to \( \gamma_{ij} \). Then \( \gamma \) is the minimum \( \gamma_{ij} \) among all node pairs.

A useful result is to note that \( \gamma \) is equal to the maximum number of link disjoint paths between nodes \( n_i \) and \( n_j \), and \( \omega_{ij} \) is equal to the maximum number of node disjoint paths between nodes \( n_i \) and \( n_j \). To evaluate network connectivity \( \omega \), the Augmentation Algorithm can be applied to the network after having it modified by splitting every node other than nodes \( n_i \) and \( n_j \) into two nodes connected by a link.

K-connectivity is another criterion for defining deterministic reliability. For a graph \( G(N,L) \), it is K-connected if it can withstand the loss of any set of K-nodes and remain connected. This implies that there exists at least K node disjoint paths between every pair of nodes. An algorithm exists to test for K-connectivity due to Klietman (1969).

### 4.2 Probabilistic Reliability

In probabilistic reliability, it is assumed that links or nodes may fail at random. Thus, failures of concern are those associated with natural failures of components, in addition to those influenced by network topology.

A well known measure to describe reliability is the two-terminal reliability, \( R(s,t) \) which gives the probability that there exists at least one path between nodes \( n_s \) and \( n_t \) (Ayoub and Shahbaz, 1996; Van Slyke and Frank, 1972; Ball and Provan, 1983; Easton and Provan, 1997; Kumamoto et al., 1997; Kumamoto et al., 1980). When the network size is relatively small, there are exact methods which compute \( R(s,t) \) through combinatorial calculations. However, these methods become impractical to use when the network size grows to be large since the needed computation takes exponential time Fishman, 1986). In this case, approximate methods including computer simulation, become the main tools of analysis to obtain numerical results for estimating \( R(s,t) \) or any other measure of reliability.

For all methods, whether combinatorial or approximate using simulation, there is the need to utilize an investigating “routine” or a search algorithm to analyze the network at hand or any of its states resulting from the network being affected by random failure. Such search algorithms try to find if nodes \( n_s \) and \( n_t \) are connected.

As we direct our attention to approximate methods, simulation becomes the main tool. The simplest simulation technique is what is known as crude Monte Carlo sampling which requires little information about the network to be analyzed for reliability estimation. Efforts to improve on the estimate obtained by crude Monte Carlo have lead to more advanced sampling techniques which utilize any available information about the network. Among such sampling techniques are the ones used to induce negative correlation between replications (Kumamoto et al., 1980), while in others, sampling plans that use permutation properties of link failures and successes were employed to gain advantage over crude Monte Carlo sampling. Karp and Luby showed how to exploit prior knowledge of the failure sets of a network to an advantage, while in Van Slyke and Frank (1972), Kumamoto et al. (1997) and Easton and Provan (1997), information about bounds on reliability were exploited.

In subsequent sections, we will review and develop appropriate algorithms for structural reliability with emphasis placed on probabilistic measures using simulation.

### 5. Operational Reliability

In the above discussion, structural reliability have raised questions of connectivity whether the network parameters are deterministic or probabilistic. As such, the reliability measures influenced by the network topology and the probability of failure, its links and nodes are designed to operate at.

As networks are designed and installed to carry traffic, and are subject in their operation to random failure, a new question of reliability, denoted as “operational reliability” becomes relevant. Since networks which cater for communication traffic fall into two categories, namely circuit switched networks or packet switched, appropriate reliability measures need to be defined.

#### 5.1 Circuit Switched Networks

For circuit switched communication networks, traffic offered or carried is usually telephone calls. Such networks are usually designed with certain probabilities of failure assigned to its links and nodes. With a network being offered, a given amount of traffic in Erlangs, and operating under conditions of random failure, a
convenient measure for operational reliability is denoted as Lost Call Traffic (LCT) (Sanso et al., 1991; Ayoub and Sutari, 1997; Hmous an Ayoub, 2003). In this respect, LCT is a generalized parameter related to the classical parameters known as blocking probability and grade of service.

When simulation is used to evaluate the LCT measure for a network under conditions of random failure, operating network states need to be considered. A network of L links, any of which could be subject to random failure, has $2^L$ network states. This is a large number for large networks, considering that the network is usually operating in few of these randomly occurring states. For a network that has links with low probability of failure, it is reasonable to assume that when link failure occurs, it affects at most one link or two links at any time. This idea will be utilized as the basis of an algorithm to evaluate the LCT at later sections.

We recognize that when circuit switched networks are loaded, congestion occur near the site of major failure, or when a link is fully loaded. Often the network adapts to such conditions through the use of alternative routing disciplines allowed for by design and by operating instructions. Therefore, the evaluation of LCT needs to take into consideration such routing disciplines.

To prepare some preliminaries to be used later to develop an algorithm to calculate the LCT by simulation, consider a network state $x_j$ with traffic offered to each link as known. When a link of capacity $c_i$ circuits is offered $a_i$ Erlangs of traffic, then from traffic theory and from Erlang’s loss formula, we have:

$$B(c, a) = \frac{a^c}{c!} \sum_{i=0}^{\infty} \frac{a^i}{i!}$$

where $B(c, a)$ is the probability of call blocking. It follows that the blocked traffic, or the LCT in Erlangs on this link is:

$$LCT = a_i B(c_i, a) \quad \ldots (5)$$

The LCT for the given network state would be to add all LCT for each link. The resulting sum when multiplied by the probability of occurrence of this state gives the LCT contribution of this state to the overall LCT for the network operating under conditions of random failure.

### 5.2 Packet Switched Networks

In packet switching, a message is divided into packets of fixed length and each packet is routed from its originating switching node to its destination node independently. Packet switching was first applied in the ARPANET, a U.S. computer network, then subsequently it was applied in commercial networks including the Internet. A packet-switched network can handle several different types of traffic concurrently. This includes high-throughput traffic like data files between computers, low-delay traffic like interactive communication between a person at a terminal and a distant computer, and real-time traffic like packetized speech and video.

#### Average Network Delay

A typical performance measure appropriate to describe operational reliability is the Average Time Delay for a single packet message. The evaluation of this delay can be achieved by extending Kleinrock’s model of a message switched network to apply to packet-switched networks (Cravis, 1981). The solution assumes that each link is a single server queue, denoting an M/M/1 queueing system with Poisson arrivals of packets, and a negative exponential distribution of packet lengths.

To solve the problem of evaluating the average network delay for a packet-switched network with offered given packet traffic subject to random failure using simulation, certain traffic and network parameters and results from queueing theory need to be defined. These results will be used later on in the development of an appropriate algorithm to evaluate the average network delay using simulation.

In the M/M/1 model, let $1/\mu$ represent the average packet length in bits, and $\lambda_i$ the total packet arrival rate at link $i$ for the mixed stream of control and data packets. Ignoring the link propagation time and the nodal processing time, we have (Cravis, 1981):

$$\lambda_i = \sum_r \sum_j y_{rj} c_i \in \pi_{rj} \quad \ldots (6)$$

Where:

- $\lambda_i$ is equal the average number of packets per second which flow on link $i$.
- $c_i$ is the capacity of link $i$ in bits per second.
- $y_{rj}$ is the packet traffic given to flow from node $r$ to node $j$.
- $\pi_{rj}$ is an appropriate route from node $r$ to node $j$ chosen to meet certain restrictions (to be defined).
The above equation shows that $\lambda_i$ is the sum of all $\gamma_{rj}$ which the route from node $r$ to node $j$ includes link $i$. We also have for the average service time, $T_i$ for link $i$:

$$T_i = \frac{1}{\mu c_i - \lambda_i}$$  \hspace{1cm} (7)

and the average time in the system $T(X_k)$ for a single packet for state $X_k$:

$$T(X_k) = \frac{1}{\gamma} \sum_{i=1}^{L_k} \frac{\lambda_i}{\mu c_i - \lambda_i}$$  \hspace{1cm} (8)

where:

$L_k$ is the number of links in state $X_k$.

$$\gamma = \sum_{r} \sum_{j} \gamma_{rj} \quad \text{with} \quad r, j = 1, 2, ..., N.$$  

Thus, $\gamma$ here represents the total offered traffic to the network.

$T(X_k)$ as given by the above equation, serves as the performance measure which can be applied to every network state resulting from simulation sampling. Averaging $T(X_k)$ over all states, we arrive at the following estimator, $T$ for the average network delay for a single packet in any given network (Li and Silvester, 1984):

$$T = \frac{\sum_{k=1}^{m} p(X_k) T(X_k)}{\sum_{k=1}^{m} p(X_k)}$$  \hspace{1cm} (9)

where:

$m$ is the number of states considered by the simulation, ( in our algorithm, it will be the most probable state).

$P(X_k)$ is the probability that state $X_k$ occurs.

6. Searching Algorithms

In this section, we consider probabilistic networks with emphasis on structural reliability. As stated before, the first step in simulation analysis for networks subject to random failure, is to sample the network state space or to generate a sample network to which a searching algorithm is applied to determine whether or not the sample network is connected. Connectivity here could be between a pair of nodes $s$ and $t$, or all pairs of nodes of $G$.

In Monte Carlo simulation, the simplest method to generate the sample state network $j$ is obtained by generating $L$ random numbers $r_i$, then for link $i$, $x_i = 0$ if $r_i < p_i$; otherwise, $x_i = 1$, and this gives the sample state vector: $X_j = \{ x_i, i = 1, 2, ..., L \}$. Since links can be in one of two states: either operating or failing, the network sampling space contains a maximum of $2^L$ states. In $K$ replications, $K < 2^L$ network states are generated and analyzed by the appropriate search algorithm, to enable the calculation of an estimate for the desired measure of performance.

To obtain better estimates with less sampling error, more advanced techniques which take advantage of any prior information about the network can be used (Kumamoto et al., 1980; Karpand Luby).

6.1 The Map Algorithm

This algorithm is capable of determining whether a network state is connected or not, and if not, the number of components it has (Ayoub and Shahbaz, 1996). Also, it determines connectivity between any pair of nodes. In this algorithm, the network graph is initially considered as one component with its links divided into two groups: the main group forming a spanning tree, and the auxiliary group containing the remaining links.

The algorithm investigates the network in two stages. In the first stage, the main group of links is searched to merge any or all the primary components found in the first stage. When the two $s$-$t$ nodes desired are found to be contained in the same component, this indicates the existence of a path between these nodes, otherwise, no such path exists. When the algorithm results in one component, then the graph is connected.

6.2 Algorithm Statements

Let $G(N,L)$ be a given network. Select a spanning tree and let its links denote the primary group numbered: $\{ l_i = i, i = 1, 2, ..., N-1 \}$. Let the remaining links denote the auxiliary group: $\{ l_i = i, i = N, N+1, ..., L \}$.

Initialization: Let $\rho = 1$, $g(l_i) = 1$, $\theta(g_i) = 1$, $i = 1$.

1- if $l_i$ is operating,
\[
g[t(l)] = g[f(l)]
\]
\[
\theta[g[f(l)]] = \theta[g[f[l]]] + 1
\]

If \(l\) is down,
\[
\rho = \rho + 1
\]
\[
g[t(l)] = g[\rho]
\]
\[
\theta[g[\rho]] = 1
\]

2- \(i = i + 1\)

If \(i < N\), go to step (1)

If \(i = N\) and \(\rho = 1\), go to step (8)

If \(i = N\) and \(\rho > 1\), go to step (3)

3- \(T[g_k] = g_k, k = 1, 2, \ldots, \rho\)

4- If \(l\) is down, go to step (6)

Otherwise:
\[
S = T[g[l]]
\]
\[
B = T[g[l]]
\]

If \(S = B\) go to step (6)

5- \(\theta[S] = \theta[S] + \theta[B]\)
\[
\theta[B] = 0
\]

For \(m = 1, 2, \ldots, \rho - 1\)

If \(T[g_m] = B\), set \(T[g_m] = S\)

\(\rho = \rho - 1\)

6- \(i = i + 1\)

If \(i \leq L\), go to step (4)

7- For nodes \(n_i, i = 1, 2, \ldots, N\)
\[
g[n_i] = T[g_i]
\]

8- Stop

9- Results:
Let final values denoted as: \(\rho_i, g_i[\cdot], \theta_i[g_i], r = 1, 2, \ldots, \rho_i\), we have:
\[
\phi_{\omega(X_i)} = 1\] if nodes \(s\) and \(t\) are in different components; otherwise, it is equal to 0.
\[
\phi(X_i) = 1\] if \(\rho_i > 1\); otherwise, it is equal to 0.

These results will be used after \(K\) replications in evaluating the measure of performance needed, as will be shown later.

### 6.3 Breadth First Search Algorithm (BFS)

Here we consider a labeling algorithm which uses the adjacency matrix to represent the network graph (Even, 1979). It is an efficient algorithm to find the shortest path between two nodes in a network, thereby establishing the presence of connectivity.

The labeling starts by assigning a distance \(D_i = \infty\) to all nodes \(i, i = 1, 2, \ldots, N\). Then selecting the source node \(s\), it is assigned a distance \(D_s = 0\). Let \(j = 1\); then for all nodes \(i\) adjacent to node \(s\), a distance \(D_i = j\) is assigned to them. If the destination node \(t\) is distance labeled, the algorithm stops; otherwise the algorithm continues to the next step.

The next step starts by choosing any labeled node \(j\) adjacent to node \(s\). If there is no such node, the algorithm stops indicating the network being disconnected. If there is such a node, it will be considered as a new source node, and the previous searching procedure will be applied to it. This continues for \(j = 2, 3, \ldots\) until node \(t\) is labeled.

#### 6.4 BFS Algorithm Statements

Given \(G(N,L)\) represented by its adjacency matrix \([a_{ij}]\). Select source node \(s\) and destination node \(t\).

1- Initialization: Assign \(D_i = \infty\) for all nodes \(i, i = 1, 2, \ldots, N\).

2- Make \(D_s = 0\)

3- Set \(i = 0\)

4- Find all unlabeled nodes \(j\) adjacent to at least one node labeled \(i\) (i.e. adjacent to the source node).

5- If no nodes are found i.e. \(a_{ij} = 0\), stop, otherwise, continue.

6- Label all nodes \(j\) found in step 4, such that: \(D_i = i + 1,\) for all \(j = 1, 2, \ldots, \rho\) and \(r = 1, 2, \ldots, \rho\), we have:
\[
\phi_{\omega(X_i)} = 1\] if nodes \(s\) and \(t\) are in different components; otherwise, it is equal to 0.
\[
\phi(X_i) = 1\] if \(\rho_i > 1\); otherwise, it is equal to 0.

Applying this algorithm to any network state \(X_j\), the result of the search is that nodes \(s\) and \(t\) are either connected or not. This gives the binary function \(\phi(X_j) = 1\) if the state is connected, and \(\phi(X_j) = 0\) if disconnected.

#### 7. Structural Reliability Algorithm (SRA)

Given a network graph \(G(N,L)\) represented by its adjacency matrix \([a_{ij}]\), with links \(l\), have a specified probability of failure \(p_i, i = 1, 2, \ldots, L\). Let \(s, t\) be any two nodes in the network, and the network is operating under conditions of random failure. To evaluate the two terminal failure probability, \(F(s,t)\) or the all terminal failure probability \(F(G)\), by simulation, the following steps for the (SRA) algorithm are used.
(a) F(s,t) evaluation
1- Generate a network state \( X_j \) by any sampling method, (for example: random number generation).
2- Apply a searching algorithm to \( X_j \) (for example: MAP or BFS) and obtain the value of \( \phi_{s,t}(X_j) \).
3- After repeating step 2 \( K \) times, then:

\[
F(s,t) = \frac{1}{K} \sum_{j=1}^{K} \phi_{s,t}(X_j)
\]

(b) F(G) evaluation
1- Generate a network state \( X_j \) as in step (a-1).
2- Apply the MAP searching algorithm to this state and obtain \( \phi(X_j) \).
3- After \( K \) replications, then

\[
F(G) = \frac{1}{K} \sum_{j=1}^{K} \phi(X_j)
\]

8. Operational Reliability Algorithms

(a) Circuit Switched Networks:
Given \( G(N,L) \) specified by its connection matrix, together with weight parameters \([C_i, p_i, i]\) for link \( i = 1, 2, ..., L \) where \( C_i \) is its capacity, and \( p_i \) its failure probability. Lost Call Traffic, LCT, as a performance measure, needs to be evaluated under conditions of random failure when the traffic load \( a_i \) offered to link \( i \), varies as a percentage of the capacity of each link \( i \).

The network could be operated using certain alternative routing when links fail. Thus, traffic on a failed link is routed through any allowed alternative route. An alternative route is typically defined to have no more than two links selected through a specified criteria. When the routed traffic is added to the actual offered link traffic, the actual offered link traffic exceeds the link’s capacity, the link operates at its full capacity, and the excess flow will be either lost or rerouted again if it is allowed for by the adopted routing algorithm.

When link failure probability \( p_i \) is small, or link availability, \( 1 - p_i \) is high, network states occurring as a result of random failure, can be those states where one link, or at most two links failing at the same time. These are the most probable states which can all be considered (Ayoub and Sutari, 1997). These states include: the no failure state, the single link failure states, and the two-link failure states. The total number of these most probable states, MPS is: MPS = 1 + L + L(L-1)/2.

Algorithm Statements

(a-1) To calculate the LCT due only to random failure:
1- Find the LCT: \( LCT(X_o) \) for the no failure state \( X_o \):

\[
LCT(X_o) = \sum_{i=1}^{L} a_i \cdot B(c_i, a_i)
\]

2- Find the LCT: \( LCT(f) \) for all failure states \( X_k, k = 1, 2, ..., MPS \) (single link failure states and two-link failure states):

\[
LCT(f) = \frac{\sum_{k=1}^{MPS} p(X_k) \cdot LCT(X_k)}{\sum_{k=1}^{MPS} p(X_k)}
\]

where,

\[
LCT(X_k) = \sum_{i=1}^{L_k} a_i \cdot B(c_i, a_i)
\]

and \( p(X_k) \) is the occurrence probability of state \( X_k \), and \( L_k \) is the number of active links in state \( X_k \).

3- Subtract the result of step 1 from the result of step 2 gives the LCT due to failure only.

(a-2) To calculate the LCT for the network when operating under conditions of random failure:

\[
LCT = \frac{\sum_{k=0}^{MPS-1} p(X_k) \cdot LCT(X_k)}{\sum_{k=0}^{MPS-1} p(X_k)}
\]

where:

\( LCT(X_o) \) is as given above.

(b) Packet Switched Networks
Given \( G(N, L) \) represented by its connection matrix \([a_{ij}], p_i, \) and \( C_i \) for every link \( i \) in \( L \), and all \( \gamma_{ij} \). The average delay per packet under conditions of random failure is calculated by the following steps:
Algorithm Statements

1- Consider the most probable states \( X_j, j = 1, 2, \ldots, MPS \), then calculate:

\[
p(X_j) = \prod_{i=1}^{L} \left( p_i + (1-2p_i) x_j \right), \quad X_j \in X
\]

2- For every node pair \( i,j \) for which \( \gamma_{ij} \neq 0 \), apply any routing algorithm to determine the alternative \( i-j \) path, or paths.

3- Using the results of step 2, link flows \( \lambda_i \) are calculated by Eq. (6), \( i = 1, 2, \ldots, L \).

4- Using the calculated link flows \( \lambda_i \), calculate \( T(X_j) \) computed using Eq. (8).

5- Compute the average network delay \( T \), using the results of steps 1 and 4 in Eq. (9).

9. Application Examples

(a) Structural Reliability

Consider the network given in Fig. 1, with \( N = 10, L = 15, p_i = 0.1, \) and \( K = 30,000 \).

Applying the Map Algorithm to this network, the following results are produced:

The probability that the network is disconnected = \( F(G) = 0.0610 \).

The \( s,t \) failure probability for all nodes, \( s \) and \( t \) = \( F(s,t) \) as shown in Table (1).

(b) Operational Reliability

Consider the packet switched network given in Fig. 2 for which the values of \( 1-p_i \) and \( \gamma_{ij} \) are given in Table (2), and that the capacity of link \( C_i = 50 \) Kbps, for all \( i \). Assume \( \gamma = 104 \).

Applying the operational reliability algorithm for packet switched networks section 8, part (b), Table (3), shows calculated results \( T_{ij} \) (seconds) for the state where link \( l_i \) has failed, and Table (4) shows some selected states when two links have failed. The final step gave for the average network delay, \( T = 0.1849 \).

10. CONCLUSIONS

Communication networks have been considered for performance analysis evaluation when under conditions of random failure using computer simulation. Circuit switched and packet switched networks have been considered, and suitable performance parameters were defined. This included parameters for structural reliability and operational reliability. In particular, Lost Call Traffic (LCT) for circuit switched networks carrying real traffic was defined and algorithms to calculate it were specified. Average network delay for packet switched networks was also defined as a performance measure, and algorithms to evaluate such parameters were given using simulation. Algorithms to analyze network states for various connectivity questions were also proposed and outlined. Finally, a generalized algorithm was developed to evaluate the appropriate performance measure for communication networks under conditions of random failure. Examples were given to illustrate the applications of these methods.

Acknowledgement

This research was supported by the University of Jordan, while the author was on sabbatical leave during the academic year 2003-2004.
Table 1

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Node No. 1 2 3 4 5 6 7 8 9

Fig. 2

Table 2: Each entry represents $\gamma_{ij}$ values

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Node 1 2 3 4 5 6 7 8 9 10
Table 3: Average time delay $T_{ij}$ for state in which link $l_{ij}$ has failed.

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<th>Failed link $l_{ij}$</th>
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Table 4: Average time delay for selected states with two link failures.

<table>
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<th>Average delay (Sec.)</th>
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REFERENCES


Karp, R., Luby, M.G. A New Monte Carlo Method for Estimating the Failure Probability of an n-Component System, Computer Science Division, University of California, Berkeley.

