Analysis of the Exchange Rates Using the ARCH Model

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ABSTRACT

In this research, we performed a deep analysis of two main variables concerning the Euro/Dollar and Yen/Dollar exchange rates (daily data that cover the period from 1-1-2003 to 31-7-2004), as well as analyzing several variables based on them. To reach our objective, we benefited from the different techniques and tests that permit us to decide about the normality, linearity, asymmetry and, particularly, the use of the evolutive skewness and kurtosis. We have used the methods associated to the homoscedastic Autoregressive models (AR model) and Autoregressive Conditional Heteroskedastic model (ARCH model). A numeric comparison between the conditional variance and the unconditional variance has clearly been achieved. We accomplished some forecastings for the August of 2004 and we estimated the forecast intervals taking into account the presence of a volatility. The quality of forecasts has been measured using the MAPE criteria. The results of our research show the importance of the applications of the AR-ARCH techniques on the time series taken in first difference. Finally, we proposed a pursuit of this research acting the technical multivariate GARCH model.

Keywords: Exchange Rate, Volatility, ARCH Model, Forecasts.

1. INTRODUCTION

Since the last three decades, the financial theory has used the time series techniques, and especially, the class of the linear ARMA processes, to analyze and forecast the financial variables, such as the exchange rates, the profit and the interest rates, and also in order to investigate the efficiency of stock market[1]. The wide application of the ARMA models covered the economic and industrial domains. However, in the financial domain, a lot of time series have a nonlinear dynamic behaviour. Indeed, for these kinds of series, we have to take into account an instantaneous variability, (named volatility), and a nonlinear dynamic[2] because assuming the linearity leads to a drop-out in the model adequate specification. It was a nonlinear generalization of the ARMA models which opens the road for dealing with the mechanisms of asymmetry (the upward and downward phases of the cycle are not symmetrical) and volatility. In fact, the observations of exchange rates are generally

approximate to one another. The adaptation of the autoregressive (AR) model is very familiar to predict the future because the predictor is linear and works by consequence, so, the calculation of the forecasts is easily achieved. However the hypothesis of a constant conditional variance is incompatible with the stylized facts; from where appears that the use of ARCH mode; seems more realistic. Engle (1982) was the first who proposed the classes of the ARCH models (Autoregressive Conditional Heteroskedastic models)[3] especially for the financial time series that have an instantaneous volatility. Indeed, for the financial time series, we observe periods of strong fluctuation (elevated variability) followed by periods of weak variability. Lots of publications tackled the ARCH models on the theoretical and practical domain. For instance, we mention Bollerslev (1986) for the GARCH models (Generalized Autoregressive Conditional Heteroskedastic models)[4], and also Engle and Bollerslev (1986) for the TARCH models (Threshold Autoregressive Conditional Heteroskedastic models), where the variance is a linear function defined by steps; and this leads to several functions of volatility that take into account the sign and the values of the shocks. Engles, Liliens and Robin (1987) suggested the ARCH-M models (Autoregressive

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Conditional Heteroskedastic models-Mean)[5]; Nelson (1990) suggested the EGARCH models (Exponential Generalized Autoregressive Conditional Heteroskedastic models). Rabemananjara and Zakoian (1991) Worked on the analysis of the asymmetric evolutions of the variance and founded the TGARCH models (Threshold Generalized Autoregressive Conditional Heteroskedastic models). Baillie, Bollerslev, and Mikkelson (1996) introduced the Fractionally Integrated Generalized Autoregressive Conditional Heteroscedastic (FIGARCH) processes. They all reported that the influence of lagged squared innovations has a slow hyperbolic rate of decay which tends to zero for long lags. They noted that this process resembles the fractionally integrated class of processes for conditional means, with similar flexibility in modelling the persistence of shocks to the conditional variance process. Using daily time series of the German Mark/US$ exchange rate, they showed that the FIGARCH (1, d, 1) model is superior to both the GARCH (1, 1) and the integrated GARCH (IGARCH (1, 1)). For more information on the different categories of the ARCH models, the reader may refer to the works of Enders (2003), Tsay (2002), Campbell, Lo and Mackinlay (1997).

Since the Paris-2 conference (November 2002), many efforts of the Lebanese government were made to manage the public debt by reducing its high cost, extending its maturity and changing its composition. Thus, the authorities that are concerned about the management of the public debt in Lebanon focus on replacing the domestic debt with external debt, particularly in US dollar and Euro. To give a simple idea of the situation of the public debt in Lebanon and the interventions exercised by the Lebanese government and the central bank for an adequate remediation, we mention some actions exercised by these authorities during the years 2003 and 2004. Indeed, it is clear that the increasing replacement of the debt in LBP by the foreign currencies, in particular in US$ and EURO, encourage the survey of the exchange rates between these different currencies. In April and May 2003, the Treasury issued bonds in foreign currencies, for an aggregate amount of US$ 622.905 million and Euro 236.250 million. Until the end of September 2003, Lebanon received US$ 2.5 billion, from the support of the participants in Paris-2 conference, which were used totally in the payment of short term debt (interest and principal) due in foreign and local currency[6]. On May 2004, as a first foreign currency

borrowing since the Paris-2 conference, the Treasury launched two issues, the first for one billion US$ and the second for 225 million Euro, for the refinancing of existing indebtedness in Lebanese pounds and/or foreign currency in the context of debt restructuring[7].

At the end of December 2004, the total debt in Lebanon stood at US$ 35.87 billion. This debt is divided in external public debt (at US$ 18.37 billion) and in domestic public debt the equivalent of US$ 17.5 billion[8]. In the context of public debt management, the central bank exchanged an equivalent amount of one billion US$ of its LBP-denominated Treasury bills portfolio against two Eurobond issues. As well, the Ministry of Finance launched, in the context of external and internal public debt management, three Eurobond issues for an aggregate amount of US$ 1.375 billion.

On the following discussion, we are going to test the presence of ARCH model in four financial time series concerning the exchange rates[9]: Opening Rate Euro/US$ (OREUUS), Opening Rate Japanese Yen/US$ (ORJYUS), Closing Rate Euro/US$ (CREUUS), Closing Rate Japanese Yen/US$ (CRJYUS)[10]. Our series cover the year 2003 and the first seven months of the year 2004, i.e. 413 observations will be considered five days per week from Monday till Friday. It is worth nothing that there is missing data (32 observations) that correspond to the closing of the Bank of Lebanon.

The aim of this study is not to consider the fundamental analysis of the exchange rates, however, it aims to analyze the volatility of the selected variables and predict their future variations. Since the empirical works provide the evidence of a covariance between the returns of the financial assets and the exchange rates[11], our research could serve also the investors in the Beirut Stock Exchange (BSE). Indeed, The BSE started its function again in January 1996 after thirteen post-closing years[12]. The choices of the exchange rates of principal foreign currencies, such as the US dollar, the Japanese Yen and the Euro, are justified by the importance of these foreign currencies in the present world. On the other hand, the recourse of the Lebanese government toward the external debt (US$ and Euro) to decrease the internal debt (high costs) requires from the responsible authorities a knowledge of the relation between dollar and Euro, in other words, the Euro/Dollar exchange rate. Unfortunately, no econometric survey is made in Lebanon between the Euro/Dollar or the Yen/Dollar. Since Japan occupies an important place in the world's

economy, a political stability in Lebanon can attract the Japanese Foreign Direct Investment (FDI) outflows and, consequently, the analysis of the volatility of Yen/Dollar exchange rate is very important. We signal that the US Dollar has reached in September 1992 the level of 3000 Lebanese Pounds (LBP), and with the arrival of the first government of the martyred president Hariri, the US dollar lowered at the end of the year 1992, to reach the level of 1835 LBP and today it is fixed at 1507.5 LBP.

The Euro is the currency of twelve European Union countries, extending from the Mediterranean to the Arctic Circle, namely Belgium, Germany, Greece, Spain, France, Ireland, Italy, Luxembourg, the Netherlands, Austria, Portugal and Finland. Euro banknotes and coins have been in circulation since 1st January 2002 and are now part of the daily life for over 300 million Europeans living in the euro area. The emergence of the Euro as a key currency, perhaps eventually rivalling the US dollar in importance, may have important macroeconomic implications for industrial as well as developing economies in the years ahead. In addition, the interactions between the major poles in the world economy, the US, Europe and Japan, can be modelled more easier. This is also true for developments affecting the region as a whole, for example the impact of an eastern enlargement of the European Union. A lot of studies tackled the modelling of the exchange rates, particularly, the relationship between euro-dollar and yen-dollar.

Clostermann and Schnatz (2000) constructed a synthetic euro-dollar exchange rate over a period from 1975 to 1998 (quarterly data). Such a synthetic real euro exchange rate has been calculated as a geometrically weighted average of the dollar exchange rates of individual European Monetary Union (EMU) currencies. The applied weights represent the share of the foreign trade of the individual euro area country to non-euro area countries in total EMU trade with non-euro area countries. According to Clostermann and Schnatz, four factors are identified as fundamental determinants of the real euro-dollar exchange rate: the international real interest rate differential, relative prices in the traded and non-traded goods sectors, the real oil price and the relative fiscal position. The euro/dollar exchange rate is estimated in a Vector Error Correction Model (VECM) based on the procedure developed by Johansen. In this study, a single equation error correction model seems to be best suited to analyze and forecast the behaviour of the

euro/dollar exchange rate in the medium-term perspective.

Camarero, Ordònez and Tamarit (2003) estimated an error correction model for the dollar real exchange rate versus seven developed countries (Germany, France, Spain, Italy, Canada, Japan and UK), using quarterly data for the the period 1970: Q1 to 1998: Q4. Brine and Safari (2003) explored the effects of the recent interventions of the American Federal Reserve and the Bank of Japan on the level and volatility of yen/dollar exchange rate; using different GARCH models including models with interventions in level. The econometric analysis achieved by Cuddington and Liang (2000) concentrates on the emergence of the EURO on the volatility of the world primary commodity prices using ARMA models with GARCH error processes. They provided evidence that episodes of internal stability of the exchange rates among the eleven countries in the European Monetary System that have adopted the Euro (Greece excluded), during 1957-98 in monthly data, were associated with periods of lower real commodity price volatility. Johnston and Scott (1999) used the mixed jump diffusion Model developed by Merton (1976), a discrete mixture of normals distribution model, and four alternative forms of the Generalized Autoregressive Conditional Heteroscedastic (GARCH) model, to analyze the daily closing spot price data for the U.S. dollar versus British pound, Canadian dollar, German mark, and Japanese yen for the period between 1978-1992. The data set, no adjustments were made for holidays, was split into five-year intervals (3817 observations) to retest all models in the to evaluate the stability of probability model over changing time periods.

Our paper is organized as follows: The introduction and a brief review of the literature are presented in section 1. In section 2, we present briefly the theoretical determination of the exchange rates. In section 3, we fill the missing data and we test the normality and the linearity of the time series. In section 4, we test the presence of the ARCH representation on the data in level. In section 5, we consider the Augmented Dickey-Fuller (ADF) tests and we apply the ARCH techniques to the data in first difference. Finally, we calculate the forecasts for August 2004 and we study their goodness using the MAPE criterion. We end our paper by conclusion and by a proposition to go further in this direction of research.

2. THEORETICAL DETERMINATION OF THE EXCHANGE RATES

During the past three decades, the instability of flexible exchange rates that is imposed in the international monetary relations was the origin and cause of rehabilitation or the apparition of several theoretical approaches. Some privilege the behaviours of arbitrage on the market of goods and services others put the monetary and financial factors forward. In this section, we limit ourselves to expose briefly the most current theoretical interpretations of the exchange rates[13]; particularly, the laws of parity, the structural analysis, the monetary approach and the equilibrium theory of portfolio.

A-The Laws of Parity

The Purchasing Power Parity (PPP) gives the relationship between exchange rates and national price levels. It is founded on the law of one price only .i.e. the goods and the services should have the same price in all countries if they were expressed in a common currency. In other words, any monetary unit could buy the same quantity of goods in its country or in all other countries after conversion in local currency. The absolute PPP states that the exchange rate is equal to the ratio of domestic to foreign prices.

According to the relative version PPP, the inflation differential at home and abroad is reflected in a corresponding change in the nominal exchange rate, i.e. the change in the exchange rates is equal to the inflation differential (it reflects the ratio of the growth rates of the index prices). Manzur (2002) investigate the relative efficacy of PPP in the measurement of international competitiveness using the Johansen cointegration and Granger causality approachs for quarterly data of exports; and using basket real exchange rates and PPP real exchange rates for the period 1973-1996, for selected East Asian countries, namely; Japan, Korea, Malaysia, Thailand and Singapore[14].

In the setting of the theory of the parity, we also mention the interest rate parity (denoted by IRP). According to this theory, the formation of exchange rates is explained by the behaviours of arbitrage between financial investments (the interest rates in real terms are similar internationally)[15]. The IRP respects the following hypotheses; the absence of transaction costs and the perfect mobility of funds. The IRP is the subject of two distinct presentations: the Covered Interest Rate Parity (CIRP) and the Uncovered Interest Rate Parity (UIRP). The CIRP is a relationship between the premium on a forward contract for foreign exchange and the differential in interest rates on securities that are identical in all respects except for the currency of denomination[16]. The UIRP establishes equality between the interest differential and the expected change in the spot exchange rate[17]. For a theoretical determination of the exchange rate, we can also mention the Fischer's equation. The fundamental idea states that the real interest rates become equal through the world because of the mobility of funds[18].

B-The Structural Analysis of the Exchange Rates

This approach tried to explain the exchange rates from the situation of the current balance of payments, or more particularly, of the commercial balance. Some works try to explain the evolution tendency of the exchange rates from the comparison between the economical structures of the countries, and more especially, of their industry, their financial insertion in the world economy and their strategy of specialization in the international division of work[19]. Williamson (1994) advanced the Fundamental Equilibrium Exchange Rates (FEER). In this approach, the real equilibrium exchange rates are derived in macroeconomic models on the assumption that the economies are simultaneously in their medium that's going to lead them to long-term internal and external equilibria.

C- Monetary Approach of the Exchange Rates

In its simplest shape, the monetary approach of the determination of the exchange rates constitutes an adapted version of the monetary approach of the balance of payments. It has been developed, then, in several directions in order to better explain the formation of the exchange rates, enhancing by this the monetary nature of the disequilibrium that appears on the markets of change.

D- The Equilibrium Theory of Portfolio

This approach has begun to take importance in the seventies of the twentieth century and it continues to occupy a primordial place in the teaching of the economy of exchange rates. It led, however, to many models that depend on the types of holdings that can be detained by the investors. In bulk, two types of interpretations are generally advanced to explain the variations of the exchange rates in the equilibrium theory of portfolio:

- For some authors, these variations are provoked by reallocations of portfolio of financial assets[20].
- For others, the variations are the result of an imperfect substitution of the national currencies[21].

Another strand of literature seeks to explain the actual trend in the exchange rate by estimating the reduced-form exchange rate equations. An example of this type is the Natural Real Exchange Rate (NATREX) model elaborated by Stein (1994, 1999), in which the regressions are derived from a macroeconomic equilibrium model.

After this theoretical skimming, we can conclude that the presentation of different explanations of the exchange rates illuminates a different aspect of the studied problem. But, it doesn't permit to obtain a homogeneous conclusions about the theoretical determination of the exchange rates. According to Salama (1989), far from the theory of anticipation, the determination of the exchange rates stays extensively unexplained. It is even legitimate to wonder, according Cartapanis (1987), on the privileged role of the anticipations in the explanation of the exchange rates. Contrary to the fundamental analysis of the exchange rates, we propose a technical analysis founded on the evolution of the time series associated to the exchange rates observed in Lebanon to predict their future variations. This type of analysis requires no knowledge of the fundamental factors of the Lebanese economy from which we will follow the evolution, but on the contrary, it requires a continual follow-up of the courses.

3. TESTING NORMALITY AND LINEARITY

First, for the four variable OREUUS, ORJYUS, CREUUS and CRJYUS that were designated by Y_i, i=1,..,4, respectively we estimated the missing observations using the simple exponential smoothing method. The effect of the missing data on the level of every variable has been measured using an indicatory variable I_t defined by:

$$I_t = \begin{vmatrix} 1 & \textit{we have a mis}\sin g \textit{ observation at time t} \\ 0 & \textit{otherwise} \end{vmatrix}$$

The estimation of the effect of the variable I_t on the variables Yi is given by (in brackets represent the statistical t):

$$Y_1 = -0.0092\ I_t + 1.1672$$
$$(-0.79)\qquad (359.9)$$

$$Y_2 = -0.0048\ I_t + 113.22$$
$$(-0.005)\qquad (428.1)$$

$$Y_3 = -0.0088\ I_t + 1.1669$$
$$(-0.76)\qquad (359.3)$$

$$Y_4 = -0.087\ I_t + 113.21$$
$$(-0.09)\qquad (426.6)$$

It seems that the variable I_t doesn't influence the variable Y_i, i=1, ..., 4. In the table (1) we present basic statistics for the four variables.

Table (1): Basic Statistics.

Variables	Average	Standard deviation	Skewness Sk	Kurtosis Ku
Y1	1.166	0.063	-0.166	-1.062
Y2	113.22	5.157	0.002	-1.634
Y3	1.166	0.063	-0.174	-1.063
Y4	113.21	5.173	0.009	-1.623

The average, the standard deviation, the Sk and the Ku all have almost the same values for the opening and closing variables. For the variables Y_2 and Y_4, the Sk approximately equals zero; and consequently, we cannot reject the null hypothesis of linearity (Similarly for Y_1 and Y_3). Since the Ku is negative (platykurtic character), we are uncertain of the normality of the four variables. We recall that the leptokurcity (leptokurtic character) of the non-gaussian distribution reflects the presence of a central part more elevated and with higher tails than those of normal law. To make the platykurtic and leptokurtic phases clear; we calculated the kurtosis and the skewness at every time. The inspection of the evolution skewness for the variables Y_1 and Y_3 reveals that there is a change of structure in 7/5/2003 followed by stabilization. Concerning the kurtosis, we note that the essential leptokurtic character is located between 14/4/2003 and 6/6/2003, the skewness and the kurtosis for the variable Y_2 and Y_4 show an interesting peak in 19/9/2003 then a stabilization having an exponential nature. We cannot therefore talk about a stable Gaussian law and a stable linear phenomenon.

3.1 Testing Normality

The test of Jarque and Bera (1984), based on the notion of skewness and kurtosis, permits to test the normality of the variables in level and in first difference. The coefficients Sk and Ku are given by:

$$Sk = \left(\frac{\mu_3}{\sigma^3}\right), \; Ku = \frac{\mu_4}{\sigma^4}$$

With μ_k and σ are respectively the centered moment of order k and the standard deviation of Y_t. The test of Jarque and Bera uses both the laws of Sk and Ku. It is given by:

$$JB = T\left(\frac{(Sk)^2}{6} + \frac{(Ku-3)^2}{24}\right)$$

If the distributions of Sk and Ku are normal then JB follows a χ^2 law with two degrees of freedom.

$$Sk \; \rightarrow \; N\left(0\,;\sqrt{\frac{6}{T}}\right) \quad Ku \; \rightarrow \; N\left(3\,;\sqrt{\frac{24}{T}}\right)$$

We construct then the statistics as follows:

$$\upsilon_1 \; = \; \frac{|Sk|}{\sqrt{\dfrac{6}{T}}}, \quad \upsilon_2 \; = \; \frac{|Ku|}{\sqrt{\dfrac{24}{T}}}$$

If the hypotheses: $H_0 : \upsilon_1 = 0$ (symmetry) and $\upsilon_2 = 0$ (normal flatness), are verified then $\upsilon_1 \prec 1.96$ and $\upsilon_2 \prec 1.96$. In the other cases, the normality hypothesis is rejected. The results of our calculation are presented in tables (2) and (3).

Table (2): Testing Normality for the Variables in Level.

Variables	υ_1	υ_2	JB	Conclusion	
Y_1	1.37	4.41	21.3	Symmetric distribution	Non-normal distribution
Y_2	0.47	6.76	45.94	Symmetric distribution	Non-normal distribution
Y_3	1.48	4.40	21.52	Symmetric distribution	Non-normal distribution
Y_4	0.42	6.76	45.34	Symmetric distribution	Non-normal distribution

The normal value at 5% is 1.96. The chi-squared value at 5% and two degrees of freedom is 5.99.

Table (3): Testing Normality for the Variables in First Difference.

Variables	υ_1	υ_2	JB	Conclusion	
ΔY_1	0.6	19.1	365.2	Symmetric distribution	Non-normal distribution
ΔY_2	1.39	14.4	209.2	Symmetric distribution	Non-normal distribution
ΔY_3	0.7	22.1	487.3	Symmetric distribution	Non-normal distribution
ΔY_4	0.77	11.2	126.0	Symmetric distribution	Non-normal distribution

3.2 Testing Linearity of Keenan

As we mentioned in our introduction, for all the phenomena characterized by mechanisms of asymmetry and strong amplitude ruptures, the class of linear ARMA model becomes maladjusted because of the existence of a nonlinear dynamics and an instantaneous variability. This maladjustment leaded to non-linear theoretical developments, in particular the category of the ARCH models. Before considering an ARCH specification, it is necessary to test the linearity of our variables. To

realize this work, we are going to use the method proposed by Keenan (1985) which consists of the following six steps:

1) We estimate an AR(p) model (p is the optimal order determined by the AIC Criterion).

$$Y_t = \varphi_0 + \sum_{i=1}^{p} \varphi_i Y_{t-i} + \varepsilon_t$$

2) We consider the model:

$$Y_t^2 = \lambda_0 + \sum_{i=1}^{p} \lambda_i Y_{t-i} + \upsilon_t$$

3) We record the residues estimated in the stages 1 and 2.

4) We regresse $\hat{\varepsilon}_t$ on $\hat{\upsilon}_t$ using the model:

$$\hat{\varepsilon}_t = \beta \hat{\upsilon}_t + e_t$$

5) We calculate the statistic as follows:

$$F = \hat{\theta}^2 \left[\frac{T - 2p - 2}{\sum_{t=p+1}^{T} \hat{\varepsilon}_t^2 - \hat{\theta}^2} \right] \quad \theta = \hat{\beta} \sqrt{\sum_{1=p+1}^{T} \hat{\upsilon}_1^2}$$

6) We test the two hypotheses:

H_0: The process Y_t is linear against the alternative hypothesis.

H_1: The process Y_t is nonlinear.

If H_0 is true, F follows the statistic of Fisher **F(1;T-2p-2)**. We have done this procedure for our four variables in level and in first difference; table (4).

Table (4): Testing Linearity of Keenan.

Variables In level	p	$\hat{\beta}$	F	Variables in first difference	p	$\hat{\beta}$	F
Y_1	7	0.4182 (130)	16 612	ΔY_1	6	-0.44978 (-0.28)	0.08
Y_2	7	0.00437 (106)	42 893	ΔY_2	6	-0.03267 (-1.24)	1.5
Y_3	7	0.4191 (131)	16 763	ΔY_3	6	-0.14082 (-0.09)	0.008
Y_4	7	0.00437 (106)	53 427	ΔY_4	6	0.00023 (0.008)	0

F(1;397) = 3.84 at 5 %.

From the table we deduce that the hypothesis of linearity for the four variables is rejected in level, and it is accepted in first difference. Therefore, the variables in first difference are symmetrical and non gaussian.

4. ARCH MODEL

The ARCH model (AutoRegressive Conditional Heteroskedastic model) has been proposed in order to analyze the financial time series. Indeed, these time series are generally characterized by volatility, i.e., the variance is a function of the time. The ARCH (q) model first introduced by Engle (1982) specifies the conditional variance of the white noise as an autoregressive dynamic model:

$$\varepsilon_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \upsilon_t$$

$$\alpha_0 \succ 0, \alpha_j \geq 0, \forall j$$

under the condition of orthogonality:

$$E\left[\varepsilon_t \middle| \varepsilon_{t-1}\right] = 0, \forall t \quad \text{where} \quad \underline{\varepsilon_{t-1}} \text{ is the history}$$

$\{\varepsilon_{t-1}, \varepsilon_{t-2}, ..., \varepsilon_0\}$. The conditional variance of the noise is given by:

$$Var\left[\varepsilon_t \middle| \varepsilon_{t-1}\right] = h_t = \sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2$$

This variance depends on the q past observations of ε_t^2. Bollerslev (1986) generalized the ARCH (q) model to the GARCH(p,q) model (Generalised AutoRegressive Conditional Heteroskedastic). The GARCH model of Bollerslev (1986) allows the conditional variance to depend upon past information; and therefore vary over time. It allows for a more flexible lag structure than the ARCH model of Engle (1982). In the GARCH model the conditional variance is predicted by past forecast errors and past variances. Formally, the model, can be expressed as follows:

$$Var\left[\varepsilon_t \middle| \varepsilon_{t-1}\right] = h_t = \sigma_t^2 = \alpha_0 + \sum_{j=1}^{q} \alpha_j \varepsilon_{t-j}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2$$

$$\alpha_0 \succ 0, \alpha_j \geq 0, \beta_j \geq 0$$

While using $\upsilon_t = \varepsilon_t^2 - Var(\varepsilon_t | \varepsilon_{t-1}) = \varepsilon_t^2 - \sigma_t^2$, we obtain an ARMA model for the squared innovations:

$$\varepsilon_t^2 = \alpha_0 + \sum_{j=1}^{Max(p,q)}(\alpha_j + \beta_j)\varepsilon_{t-j}^2 + \upsilon_t - \sum_{j=1}^{p}\beta_j\upsilon_{t-j}$$

It is clear that in the GARCH(p,q) representation, p is the order of the moving average part and q is the order of the autoregressive part. Taking the expectation of each of the two members, we get:

$$E(\varepsilon_t^2 | \underline{\varepsilon_{t-1}}) = \alpha_0 + \sum_{j=1}^{Max(p,q)}(\alpha_j + \beta_j)E(\varepsilon_{t-j}^2)$$

The sequence of the conditional variances satisfies a linear recurrence equation. Therefore, if we let

$$\alpha(B) = \sum_{j=1}^{q}\alpha_j B^j, \ \beta(B) = \sum_{j=1}^{p}\beta_j B^j$$

with B as the backshift operator, we get:

$$[1 - (\alpha(B) + \beta(B))]\varepsilon_t^2 = \alpha_0 + [1 - \beta(B)]\upsilon_t$$

The process ε_t^2 is asymptotically weak stationary if the roots of the polynomial

$$1 - \sum_{j=1}^{max(p,q)}(\alpha_j + \beta_j)B^j$$

are located outside the circle unit. It is equivalent to (see Gourieroux 1994):

$$1 - \sum_{j=1}^{max(p,q)}(\alpha_j + \beta_j) \succ 0$$

In such a case, the variance of ε_t is independent of the time (necessary and sufficient condition of a weak stationary of ε_t). The unconditional variance of ε_t becomes:

$$\sigma^2 = \alpha_0(1 - \alpha(1) - \beta(1))^{-1}$$

While imposing the relation

$$\varepsilon_t = \sigma_t Z_t; \ E(Z_t) = 0, \ Var\ (Z_t) = 1$$

and since Z_t is independent of the past of ε_t, Nelson (1990) proposed the following GARCH(p,q) model:

$$\sigma_t^2 = \alpha_0 + \sum_{j=1}^{Max(p,q)}(\alpha_j Z_{t-j}^2 + \beta_j)\sigma_{t-j}^2$$

We note that the estimation method of the GARCH model is made by the BFGS [22] procedure while using an initial estimation according to the simplex technique.

4.1 Application of the ARCH Model on the Variables in Level

We have seen that a GARCH(p,q) model is assimilated to an ARMA model for the squared innovations. The construction of a GARCH model respects the following steps:

1) We estimate an AR(p) model for the process Y_t with p as the optimal order determined by the AIC criteria.
2) We record the residues of the AR(p) model. The $\hat{\varepsilon}_t^2$ help in identifying the p and q orders for the GARCH model by calculating the estimated autocorrelation $\hat{\rho}(k)$:

$$\hat{\rho}(k) = \frac{\sum_{t=k+1}^{T}(\hat{\varepsilon}_t^2 - \hat{\sigma}^2)(\hat{\varepsilon}_{t-k}^2 - \hat{\sigma}^2)}{\sum_{t=1}^{T}(\hat{\varepsilon}_t^2 - \hat{\sigma}^2)^2}$$

$$\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t^2$$

McLeod and Li (1983) showed that the asymptotic variance of $\hat{\rho}(k)$ is T^{-1}. We can also use the Ljung-Box Q statistics:

$$Q = T(T+2)\sum_{k=1}^{M}\frac{\hat{\rho}^2(k)}{T-k}$$

If the $\hat{\varepsilon}_t^2$ are independent, then Q follows a chi square distribution at M degrees of freedom (in what follows, several values of M will be taken into account). If Q is less than a certain critical value of χ^2, then we accept the null hypothesis which states that the ARCH representation does not exist. Engle (1982) suggested the test (Lagrange Multiplier LM) to test the hypothesis of ARCH(q) by considering the regression:

$$\varepsilon_t^2 = \theta_0 + \sum_{j=1}^{q}\theta_j\varepsilon_{t-j}^2 + \eta_t$$

The statistic $LM = TR^2$ follows χ_q^2 with R^2 is the coefficient of determination of the regression.

4.1.1 Modeling for the Variable Y_{1t}

The AIC criterion permits to propose an AR(7) model as follows:

$$Y_{1t} = 0.019 + 0.667 Y_{1t-1} + 0.269 Y_{1t-2} - 0.112 Y_{1t-4} + 0.312 Y_{1t-5} - 0.152 Y_{1t-7}$$

$$(2.0) \quad (14.2) \quad\quad (4.9) \quad\quad (-2.1) \quad\quad (5.8) \quad\quad (-3.8)$$

and $\hat{\sigma}^2 \approx 0.0001$. The squared errors follow an AR(5) model:

$$\varepsilon_{1t}^2 = 0.00005 + 0.119\,\varepsilon_{1t-1}^2 + 0.173\,\varepsilon_{1t-4}^2 + 0.240\,\varepsilon_{1t-5}^2$$

$$(4.1) \quad\quad (2.5) \quad\quad\quad (3.6) \quad\quad\quad (4.9)$$

$$LM = 125.4, \quad \chi_{5;0.05}^2 = 11.1$$

This leads to the rejection of the null hypothesis (independence between the ε_{1t}^2). It seems that probably there is a ARCH(q) or GARCH (p,q) representation.

4.1.2 AR(7)-ARCH(5) model for the variable Y_{1t}

$$Y_{1t} = 0.016 + 0.738\,Y_{1t-1} + 0.25\,Y_{1t-2} - 0.1352\,Y_{1t-4} + 0.247\,Y_{1t-5} - 0.113\,Y_{1t-7}$$

$$(1.95) \quad (22.8) \quad\quad (10.26) \quad\quad (-12.02) \quad\quad (7.87) \quad\quad (-2.98)$$

$$h_t = \sigma_t^2 = 0.000067 + 0.3485\,\varepsilon_{t-5}^2$$

$$(8.69) \quad\quad (3.54)$$

The other parameters of the ARCH model are statistically zero. The estimation of the marginal variance of the AR-ARCH model is:

$$\overline{E(\sigma_t^2)} = \frac{0.000067}{1 - 0.3485} \approx 0.0001$$

It is equal to the value observed from the AR(7) model.

Modeling for the Variable Y_{2t}

The AIC criterion proposes an AR(7) model as follows:

$$Y_{2t} = 1.258 + 0.777 Y_{2(t-1)} + 0.167 Y_{2(t-2)} + 0.259 Y_{2(t-5)} - 0.1212 Y_{2(t-6)} - 0.094 Y_{2(t-7)}$$

$$(1.6) \quad (15.8) \quad\quad (3.1) \quad\quad\quad (4.8) \quad\quad\quad (-2.0) \quad\quad (-1.9)$$

and $\hat{\sigma}^2 \approx 0.5358$.

The squared errors follow an AR(1) model and then we estimate the following AR(7)-ARCH(1) model as follows:

$$Y_{2t} = 1.375 + 0.809 Y_{2(t-1)} + 0.144 Y_{2(t-2)} + 0.214 Y_{2(t-5)} - 0.111 Y_{2(t-6)} - 0.068 Y_{2(t-7)}$$

$$(2.14) \quad (20.5) \quad\quad (7.4) \quad\quad\quad (15.4) \quad\quad\quad (-29.8) \quad\quad (-16.8)$$

$$h_t = \sigma_t^2 = 0.445 + 0.14\varepsilon_{t-1}^2$$

$$(11.8) \quad\quad (2.5)$$

The estimation of the marginal variance of the ARCH(1) model is:

$$\overline{E(\sigma_t^2)} = \frac{0.445}{1-0.14} = \frac{0.446}{0.86} = 0.5174$$

For the AR(7) model, the estimation of the variance is 0.5358; consequently, the AR(7)-ARCH(1) specification induces a decrease in marginal variance by 3.56%.

4.1.3 Modeling for the Variable $U_1(t)$ and $U_2(t)$

$U_1(t)$ (respectively $U_2(t)$) is the difference between Euro/Dollar (respectively Yen/Dollar) closing exchange rate at the time t and Euro/Dollar (respectively Yen/Dollar) opening exchange rate at the same time. The behaviors of the ACF and PACF of these two variables allows us to consider them like a random walk. The Ljung-Box Q statistics are presented in table (5).

4.1.4 Modelling for the Variable $V_1(t)$ and $V_2(t)$

Let $V_1(t)$ (respectively $V_2(t)$) be the difference between Euro/Dollar (respectively Yen/Dollar) closing exchange rate at the time t and Euro/Dollar (respectively Yen/Dollar) opening exchange rate at the time t+1. we defined these two variables in order to know if the closing exchange rate at the time t has a certain influence

on the opening exchange rate a the time t+1. We don't consider here the notion of Granger's causality but rather we try to answer the following question: Is the difference between the closing and opening exchange rates baffled one day a random walk?

Table (5): Ljung-Box Q Statistics.

Degree of freedom	8	16	24	32	40
Q for $U_1(t)$	2.14	8.0	17.08	26.0	33.87
Q for $U_2(t)$	5.41	9.03	17.84	27.19	33.92

The Ljung-Box Q statistics of the errors are presented in table (6).

Table (6): Ljung-Box Q Statistics of the Errors.

Degree of freedom	8	16	24	32	40
Q	4.0	9.2	25.4	37.8	43.5

The behaviours of the ACF and PACF of the variable $V_1(t)$ propose an AR(6) model. By eliminating the non significant parameters and keeping the parameter 2 (its deletion leads to a certain deterioration of the statistical Q), we estimate the following AR(6) model:

Model M_1:

$$V_{lt} = -0.326V_{l(t-1)} - 0.079V_{l(t-2)} - 0.153V_{l(t-4)} + 0.173V_{l(t-5)} + 0.112V_{l(t-6)}$$

$$(-6.6) \qquad (-1.6) \qquad (-3.2) \qquad (3.4) \qquad (2.3)$$

and $\hat{\sigma}^2 \approx 0.9 \times 10^{-4}$.

The squared errors follow an AR(5) model as follows:

$$e_t^2 = 0.00004 + 0.0892e_{t-1}^2 + 0.1308e_{t-4}^2 + 0.3406e_{t-5}^2$$

$$(3.6) \qquad (1.9) \qquad (2.8) \qquad (7.2)$$

LM =123.8. Let's try the representation ARCH(5) or GARCH(1,1):

Model M_2:

$$V_{lt} = -0.220V_{l(t-1)} - 0.136V_{l(t-4)} + 0.123V_{l(t-5)} + 0.094V_{l(t-6)}$$

$$(-5.3) \qquad (-3.2) \qquad (2.2) \qquad (2.3)$$

$$h_t = \sigma_t^2 = 0.00006 + 0.360\varepsilon_{t-5}^2$$

$$(10.2) \qquad (4.0)$$

$$\overline{E(\sigma_t^2)} = \frac{0.00006}{1-0.36} = 0.000094.$$

Which is practically equal to the value obtained from the AR(6) model. Let's try GARCH(1,1) model:

Model M$_3$:

$$V_{1t} = -0.278\, V_{1(t-1)} - 0.137\, V_{1(t-4)} + 0.134\, V_{1(t-5)} + 0.104\, V_{1(t-6)}$$

$$\quad\quad\ (-5.3)\quad\quad\ (-2.8)\quad\quad\ (2.1)\quad\quad\ (2.1)$$

$$h_t = \sigma_t^2 = 0.0000096 + 0.109\varepsilon_{t-1}^2 + 0.785\sigma_{t-1}^2$$

$$\quad\quad (2.2)\quad\quad (3.1)\quad\quad (11.7)$$

Similarly, the GARCH (1,1) specification doesn't succeeded in reducing the variance of the errors in the AR(6) model. Consider now the variable $V_2(t)$. The ACF and PACF of this variable permit to estimate the following model:

Model M$_4$ [23]:

$$V_{2(t)} = -0.215 V_{2(t-1)} - 0.088 V_{2(t-3)} - 0.162 V_{2(t-4)} + 0.123 V_{2(t-5)}$$

$$\quad\quad\ (-4.5)\quad\quad\ (-1.8)\quad\quad\ (-3.3)\quad\quad\ (2.5)$$

$$\hat{\sigma}^2 \approx 0.47 \text{ and LM} = 42.5.$$

The squared residues of the above estimated model behave like a white noise; and consequently, we don't consider an ARCH specification or GARCH. The Ljung-Box Q statistics of the squared residues are presented in table (7).

Table (7): Ljung-Box Q Statistics of the Squared Residues.

Degree of freedom	8	16	24	32	40
Q	7.6	10.8	17.3	19.5	24.3

5. TESTS OF AUGMENTED DICKEY FULLER

The first aim of testing unit roots is to discover the nature of the non- stationarity in macroeconomic time series in order to determine their order of integration. Indeed, the specification of the nature of the type of trend (deterministic or stochastic) is fundamental in all analysis of the time series and especially for the forecasts. In the literature of the applied econometrics, there is a large bibliography of the different procedures that deal with the order of the integration in a time series. For example, we mention Fuller (1976), Dickey and Fuller (1979, 1981), Phillips and Stairway (1987, 1988), Phillips (1987), Sargan and Bhargava(1983), Dickey and said (1984) and Ertur (1992). The results gotten by Nelson and Plosser (1982) accentuated the distinction between the TS time series (stationary around the trend) and DS time series (stationary after a first difference)[24].

TS Time Series

Let's consider an example of the X_t process; consider $X_t = \alpha + \beta t + \varepsilon_t$. It is clear that $E(X_t) = \alpha + \beta t = f(t)$ and $V(X_t) = \sigma^2$.

Ds Time Series

Let's suppose that $X_t - X_{t-1} = \varepsilon_t$. We have easily: $E(X_t) = X_0$ and $V(X_t) = \sigma^2 t$. In the two cases, the process X_t is non-stationary but the cause of the non-stationarity is different. The DF (Dickey-Fuller) procedure considers an AR(1) model: $X_t = \varphi X_{t-1} + \varepsilon_t$ with X_0 is fixed and ε_t is a white noise. The DF procedure proposes a one-sided test on the left, while testing the null hypothesis:

H_0: $\varphi=1$ (X_t is random walk)

H_a: $|\varphi| < 1$ (X_t is asymptotically stationary)

In the DF procedure, three types of models are proposed:

(M_1) $X_t = \varphi X_{t-1} + \varepsilon_t$ AR(1) model without constant

(M_2) $X_t = \mu + \varphi X_{t-1} + \varepsilon_t$ AR(1) model with constant

(M_3) $X_t = \varphi X_{t-1} + bt + c + \varepsilon_t$ AR(1) model with deterministic trend.

If the null hypothesis H0 is verified, then X_t is non-stationary. Let's write the models above as the following:

(DF_1) $\Delta X_t = (\varphi-1)X_{t-1} + \varepsilon_t$

(DF_2) $\Delta X_t = \mu + (\varphi-1)X_{t-1} + \varepsilon_t$

(DF_3) $\Delta X_t = (\varphi-1)X_{t-1} + bt + c + \varepsilon_t$

Therefore, the null hypothesis becomes **H0: $\varphi=1$** or **$\rho=\varphi-1=0$**. With the help of Monte-Carlo simulations, Dickey and Fuller tabulated the critical values for samples of different sizes. These tables are analogous to

the tables of the t Student[25]. Practically, we estimate the parameter $\boldsymbol{\rho}$ using OLS method. If the statistic $t_{\hat{\rho}} > t_{crit}$ then we accept H_0. Since the above models

are limited to the AR(1) processes, Dickey and Fuller generalized their procedure for an AR(p) process using the ADF (Augmented Dickey Fuller) procedure. The ADF procedure proposes three types of models:

$$\left(ADF_1\right)\quad \Delta X_t = \rho X_{t-1} + \sum_{j=2}^{p} \Delta X_{t-j+1} + \varepsilon_t$$

$$\left(ADF_2\right)\quad \Delta X_t = \mu + \rho X_{t-1} + \sum_{j=2}^{p} \Delta X_{t-j+1} + \varepsilon_t$$

$$\left(ADF_3\right)\quad \Delta X_t = \rho X_{t-1} + \sum_{j=2}^{p} \Delta X_{t-j+1} + bt + c + \varepsilon_t$$

The critical values of DF and ADF statistics are similar.

5.1 Modelling for the Variable $\Delta Y_{1t} = Z_{1t}$

The AIC criterion proposes an AR(6) model; consequently we estimate the following:

$$Z_{1t} = -0.288 Z_{1(t-1)} - 0.129 Z_{1(t-4)} + 0.207 Z_{1(t-5)} + 0.119 Z_{1(t-6)}$$

$$(-6.0) \qquad\qquad (-2.7) \qquad\qquad (4.1) \qquad\qquad (2.5)$$

$$\hat{\sigma}^2 = 0.0001$$

The squared errors follow an AR(5) model:

$$\varepsilon_t^2 = 0.00005 + 0.108\,\varepsilon_{t-1}^2 + 0.190\,\varepsilon_{t-4}^2 + 0.261\,\varepsilon_{t-5}^2$$

$$(3.9) \qquad (2.3) \qquad\quad (4.0) \qquad\quad (5.4)$$

$$LM = 130.7 > \chi_{5;0.05}^2 = 11.1$$

Table (8): Augmented Dickey-Fuller (ADF) Statistics.

Variables	Models			Conclusion
	ADF_1	ADF_2	ADF_3	
Y_1	$\hat{\rho} = 0.0004$ (0.82)	$\hat{\rho} = -0.0157$ (−1.95) $\hat{\mu} = 0.0188$ (1.99)	$\hat{\rho} = -0.0323$ (−2.17) $\hat{c} = 0.036$ (2.24) $\hat{b} = 0.00001$ (1.32)	DS time series
Y_2	$\hat{\rho} = -0.0002$ (−0.60)	$\hat{\rho} = -0.0113$ (−1.59) $\hat{\mu} = 1.263$ (1.56)	$\hat{\rho} = -0.0247$ (−1.95) $\hat{c} = 2.93$ (1.91) $\hat{b} = -0.0007$ (−1.28)	DS time series
Y_{3t}-Y_{1t}	$\hat{\rho} = -1.0956$ (−8.11)	$\hat{\rho} = -1.1326$ (−8.26) $\hat{\mu} = -0.000026$ (−1.5)	$\hat{\rho} = -1.133$ (−8.25) $\hat{c} = -0.0003$ (−0.85) $\hat{b} = 0.0000002$ (0.13)	Stationary time series

Y_{4t}-Y_{2t}	$\hat{\rho} = -0.903$ (-7.19)	$\hat{\rho} = -0.923$ (-7.27) $\hat{\mu} = -0.0169$ (-1.04)	$\hat{\rho} = -0.923$ (-7.26) $\hat{c} = -0.0276$ (-0.83) $\hat{b} = 0.00005$ (0.37)	Stationary time series
Y_{3t}-$Y_{1(t+1)}$	$\hat{\rho} = -1.445$ (-7.87)	$\hat{\rho} = -1.538$ (-8.12) $\hat{\mu} = -0.0009$ (-1.92)	$\hat{\rho} = -1.57$ (-8.22) $\hat{c} = -0.002$ (-2.0) $\hat{b} = 0.000005$ (1.21)	Stationary time series
Y_{4t}-$Y_{2(t+1)}$	$\hat{\rho} = -1.483$ (-8.23)	$\hat{\rho} = -1.483$ (-8.22) $\hat{\mu} = 0.0015$ (0.045)	$\hat{\rho} = -1.492$ (-8.24) $\hat{c} = 0.044$ (0.63) $\hat{b} = -0.0002$ (-0.69)	Stationary time series

We reject the null hypothesis (independence of ε_t^2) and we propose an ARCH(q) or GARCH(p,q) representation. In what follows, we estimate an AR(5)-ARCH(5) model and another AR(5)-GARCH(1,1) model:

AR(6)-ARCH(5) model

$$Z_{1t} = -0.22\,Z_{1(t-1)} - 0.093\,Z_{1(t-4)} + 0.148\,Z_{1(t-5)} + 0.084\,Z_{1(t-6)}$$
$$\quad(-11.2)\qquad(-3.2)\qquad\quad(4.8)\qquad\qquad(3.7)$$
$$h_t = \sigma_t^2 = 0.000061 + 0.049\,\varepsilon_{t-4}^2 + 0.367\,\varepsilon_{t-5}^2$$
$$\qquad\quad(12.8)\qquad(1.5)\qquad\quad(5.3)$$

AR(6)-GARCH(1,1) model

$$Z_{1t} = -0.26\,Z_{1(t-1)} - 0.097\,Z_{1(t-4)} + 0.16\,Z_{1(t-5)} + 0.091\,Z_{1(t-6)}$$
$$\quad(-4.75)\qquad\quad(-1.98)\qquad\quad(3.16)\qquad\quad(1.82)$$
$$h_t = \sigma_t^2 = 0.00001 + 0.107\,\varepsilon_{t-1}^2 + 0.789\,h_{t-1}$$
$$\qquad\quad(1.9)\qquad(2.8)\qquad\quad(10.6)$$

The estimation of the non-conditional variance of the GARCH (1, 1) model is $\overline{E(\sigma_t^2)} = 0.9615 \times 10^{-4}$. For the above AR(6) model, the variance decreases by 4 %.

AR(27) model

Inspecting the behaviours of ACF and PACF for the residuals in the AR(6) model, we propose the AR(27) model as follows:

$$Z_{1t} = -0.291Z_{1(t-1)} - 0.128Z_{1(t-4)} + 0.219Z_{1(t-5)} + 0.135Z_{1(t-6)} + 0.12Z_{1(t-17)}$$
$$\quad(-6.1)\qquad\quad(-2.7)\qquad\quad(4.3)\qquad\quad(2.7)\qquad\quad(2.5)$$

$$\hat{\sigma}^2 = 0.996 \times 10^{-4}$$

$$-0.129Z_{1(t-21)} - 0.146Z_{1(t-22)} + 0.168Z_{1(t-27)}$$
$$\quad(-2.7)\qquad\qquad(-2.8)\qquad\qquad(3.6)$$

AR(27)-ARCH(5) model

The squared errors follow an AR(5) model, therefore we estimate the following model:

$$Z_{1t} = -0.225Z_{1(t-1)} - 0.106Z_{1(t-4)} + 0.154Z_{1(t-5)} + 0.090Z_{1(t-6)} +$$

$$(-5.2) \qquad\qquad (-2.5) \qquad\qquad (3.0) \qquad\qquad (2.2)$$

$$0.149Z_{1(t-17)} - 0.119Z_{1(t-21)} - 0.127Z_{1(t-22)} + 0.126Z_{1(t-27)}$$

$$(3.4) \qquad\qquad (-2.7) \qquad\qquad (-3.0) \qquad\qquad (3.1)$$

$$\sigma_t^2 = 0.000064 + 0.312\varepsilon_{t-5}^2$$

$$(9.8) \qquad (3.8)$$

$$\overline{E(\sigma_t^2)} = 0.93 \times 10^{-4} < \hat{\sigma}^2 = 0.996 \times 10^{-4}$$

(a reduction of 7%).

Modelling for the Variable $\Delta Y_{2t} = Z_{2t}$

Using the AIC Criterion, we identify the AR(6) model:

$$Z_{2t} = -0.212Z_{2(t-1)} - 0.088Z_{2(t-4)} + 0.205Z_{2(t-5)} + 0.098Z_{2(t-6)} \qquad \hat{\sigma}^2 = 0.5358$$

$$(-4.4) \qquad\qquad (-1.8) \qquad\qquad (4.2) \qquad\qquad (2.0)$$

The squared errors follow an AR(1) model:

$$\varepsilon_t^2 = 0.397 + 0.252\varepsilon_{t-1}^2$$

$$(6.8) \qquad (5.2)$$

$$LM = 97.7 > \chi_{1;0.05}^2 = 3.84$$

Accordingly, We reject the null hypothesis (independence of ε_t^2) and we propose an ARCH(1) representation.

AR (6)-ARCH (1) model

$$Z_{2t} = -0.168Z_{2(t-1)} - 0.063Z_{2(t-4)} + 0.180Z_{2(t-5)} + 0.075Z_{2(t-6)}$$

$$(-3.4) \qquad\qquad (-1.4) \qquad\qquad (3.9) \qquad\qquad (1.7)$$

$$h_t = \sigma_t^2 = 0.4515 + 0.134\varepsilon_{t-1}^2$$

$$(12.5) \qquad (2.5)$$

$$\overline{E(\sigma_t^2)} = 0.5214 < \hat{\sigma}^2 = 0.5358$$

(a reduction of 2.76%).

6. CALCULATION OF THE FORECASTS FOR THE DIFFERENT MODELS

In an ARMA-GARCH modelling, we first estimate the autoregressive and moving average parameters of the ARMA model, then, we calculate some forecasts at horizon 1 of the Y_t. We get:

$$\hat{Y}_t = \left[\frac{\hat{\phi}(B)}{\hat{\theta}(B)} - 1\right]Y_t$$

$\phi(B)$ Is the autoregressive polynomial and $\theta(B)$ is the moving average polynomial. The variability is estimated by:

$$\hat{\sigma}^2 = \frac{1}{T}\sum_{t=1}^{T}\hat{\varepsilon}_t^2$$

Regardless of the effect of estimation of $\hat{\phi}(B)$ and $\hat{\theta}(B)$, the shape of the forecasting intervals becomes $\left[\hat{Y}_t \pm 2\hat{\sigma}\right]$, and therefore, the forecast intervals have the same width. Second, we estimate the ARMA-GARCH model and calculate forecasts of horizon 1. We get

$$\hat{\hat{Y}}_t = \left[\frac{\hat{\hat{\phi}}(B)}{\hat{\hat{\theta}}(B)} - 1\right]Y_t$$

The forecast intervals have the shape $\left[\hat{Y}_t \pm 2\hat{h}_t \right]$, and the width of these intervals depends on the time t. To measure the goodness of forecasts for August (22 workdays), we used the MAPE criterion given by:

$$MAPE = \frac{1}{22} \sum_{h=1}^{22} \frac{\left| \hat{Y}_{T+h} - Y_{T+h} \right|}{Y_{T+h}}$$

$\hat{Y}_{T+h}$ =predictionction at Horizon h and Y_{T+h} = real value at the time T+h.

Table (9): Goodness of Forecasts: MAPE Criterion.

Variables in level			
Y_1	AR(7) Model: 0.475%	AR(7)-ARCH(1) model: 0.473%	
Y_2	AR(7) Model: 0.537%	AR(7)-ARCH(5) model: 0.545 %	
Variables in First Difference			
ΔY_{1t}	AR(6) Model: 1.63%	AR(27) model 1.98 %	AR(27)-ARCH(5) model: 0.552%
ΔY_{2t}	AR(6) Model: 1.68%	AR(6)-ARCH(1) model: 0.543%	

To complete the forecast analysis, we calculated the forecast intervals for the variables Y_{1t} and Y_{2t} taken in level and in first difference. The inspection of these diagrams, (Figures from (9) to (12)), shows that for the variable Y_{1t} (in level and in first difference), the volatility is nearly zero; therefore, we can say that we are in homoskedastic situation. For the variable Y_{2t}, it is clear that the volatility varies from period to period, which is revealed by different width of the forecast intervals. Finally, considering the results of testing Dickey-Fuller, the values of MAPE and the behaviour of the volatility, we can keep the variables in first difference using the AR-ARCH approach.

7. CONCLUSION

In this paper, the obtained results are the following results:

1- The kurtosis and the skewness are practically the same for the opening and the closing exchange rates for EURO/DOLLAR and YEN/ DOLLAR. The variables in level have the symmetrical, nonlinear and non-normal distributions. On the other hand, for the variables in first difference, we can keep symmetrical, linear and non-Gaussian distributions.

2- The Augmented Dickey-Fuller (ADF) statistics allow us to get two types of variables: The variables Y_{1t} and Y_{2t} are DS time series. Yet the other variables $Y_{3t} - Y_{1t}$, $Y_{4t} - Y_{2t}$, $Y_{3t} - Y_{1(t+1)}$ and $Y_{4t} - Y_{2(t+1)}$ are weakly stationary. Besides, the variables $Y_{3t} - Y_{1t}$ and $Y_{4t} - Y_{2t}$ are random walks.

On the other hand, the variables $Y_{3t} - Y_{1(t+1)}$ and $Y_{4t} - Y_{2(t+1)}$ are identified respectively by AR (6) and AR (5).

3- The centers of the forecast intervals are practically the same with the two approaches AR and AR-ARCH for the variables in level and in first difference. But, the quality of forecasts is better for the variables in first difference and in AR-ARCH representation (The value of MAPE is less than or equal to 0.5%).

4- The volatility is more important in the opening Yen/Dollar exchange rate than in the Euro/Dollar case.

Finally, the modelling of the volatility of the Euro/Dollar and Yen/Dollar exchange rates using AR-ARCH model can help to decrease the risk in the financial decisions that are in relation with the exchange rates. Besides, a certain security at short run is added to the investors in Beirut Stock Exchange and in the other financial institutions, notably the central bank, the commercial banks and the societies of credits.

Suggestion: To go farther, first, we propose a multivariate GARCH model. We can estimate a GARCH (1, 1) on a two-variable system of exchange rates. Second, we elaborate a bivariate error correction model (BECM) for the Euro/Dollar and Yen/Dollar exchange rates to identify the long-run equilibrium. Such a long-run relationship can help the Lebanese government to manage the public debt by reducing its high cost.

NOTES

(1)   See Box and Jenkins (1976), Granger and Morgenstern (1970).

(2)   One mentions the two important works of Priestley (1988) and Tong (1990).

(3)   For an exhaustive preview of the ARCH models, see Bollerslev, Chou and Kroner (1992).

(4)   Bollerslev studied the variable

$$\pi_t = 100 Log\left(\frac{GD_t}{GD_{t-1}}\right) \text{ where } GD_t \text{ is the}$$

general level of prices in the time t for the United States, and on quarterly data covering the period 1948-Q2; 1983-Q4. He estimated an AR (4) models for the variable π_t then an AR (4)-GARCH (1, 1) model for the same data.

(5)   We mention the research realised by Chou (1988). Chou uses the GARCH (1, 1) M model to analyze the weekly returns of the New York Stock Exchange (NYSE).

(6)   Bank of Lebanon, Q2-2003, P.22 and Q3-2003, P. 20.

(7)   Bank of Lebanon, Q2-2004, P. 20.

(8)   Bank of Lebanon, Q4-2004, P. 21-22.

(9)   Source: Quarterly Bulletin, Bank of Lebanon.

(10) This is an indirect calculation: If the Yen/DOLLAR exchange rate is 125, it means that $1 equals 125Yen.

(11) See Wolff (1987).

(12) The yearly report of the Lebanese Bank (1996). We denote that the value of traded shares on the BSE is given in million of US$.

(13) For a complete presentation, see for example Dehove (1985), Krueger (1985), Badran (1992) Allegre (1997) and Boutrguinat (1999).

(14) See Manzur (2002), Chapter 4, P.79. The cost of the basket in US dollars is given by

$$\sum_{i=1}^{n} \frac{x_i}{S_{it}} \text{ Where } S_{it} \text{ is the Cost of \$US1 at time t in}$$

terms of the currency of county i (i=1... n), and x_i represents the quantity of units of currency i.

(15) See Izan (2002), Chapter 5, P. 97.

(16) See Frenkel and Levich (1975).

(17) See Edison and Paul (1993).

(18) See Gagnon and Unferth (1995).

(19) Aglietta, Orlean and Oudiz (1980).

(20) For a synthesis of this type of works, see Aftalion and Losq (1985).

(21) See Salama (1989).

(22) BFGS proposed by Broyden, Fletcher, Goldfarb and Shanno. It is described in Press, et al. (1988).

(23) For the four models Mi, i = 1, 2, 3 and 4, we calculated the forecast errors for the august month (22 workdays). We got the values -0.00099, -0.00094, -0.00097 and 0.2599, respectively.

(24) See Dickey et al (1986), Phillips (1987), Schmidt (1990), Mourad 2002.

(25) See Bresson and Pirotte (1995), p. 418-431.

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