Dirasat, Administrative Sciences, Volume **37**, No. **1**, 2010

Cost-Benefit Analysis of the Marginal Revenue from Indirect Taxes

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ABSTRACT

The paper summarizes the literature on the marginal cost of tax revenues and develops a formula for its measurement in terms of estimable demand parameters. The paper dwells on the paper by Diamond and Mirrlees (1971), inter alia, where the demand interaction between public goods and the taxed goods is taken into consideration. The interaction term is substituted out using Euler's theorem and the Slutsky equation under the assumption that demand functions are homogeneous of degree zero. The paper illustrates the cost-benefit calculation of marginal tax revenues from indirect taxes using actual data from British studies and compares this with results obtained by other authors in their similar studies. The paper concludes that the Benefit-Cost ratio of marginal tax revenues might exceed one and analysts should be cautioned against any assumptions regarding this ratio of being less than one. The paper, also, highlights some areas for further research to be carried out along the same lines of analysis.

Keywords: Marginal Cost, Indirect Taxes, Public Good.

1. INTRODUCTION

A number of definitions and measures of marginal tax revenues have appeared in the literature. The issues raised justify increased attention because of the renewed interest in excess burden and its links to the marginal cost of indirect tax revenues and thus to cost-benefit analysis [see Topham (1984,1985), Pauwels (1986), Browning (1989) and Triest (1990)]. Some are set in a general framework; others concern themselves with exact and correct measures of excess burden; and some papers stress that the marginal costs of indirect tax revenues can only be calculated accurately when we know from where the public funds have been derived and to which uses they will be put. This latter notion leads on to the consideration of both sides of the budget and considers the interactions between the public services provided and the tax base; for example, road provision financed by a tax on cars is likely to have an effect on the demand for cars through price effects and because of any stimulus to demand arising from better roads provided. One of the purposes of this paper is to demonstrate how these interactions can be calculated in terms of easily estimated behavioral parameters.

The issues raised by this debate are not trivial. Calculations of the marginal cost of indirect tax revenues (MCT) vary from 1.07 to 1.21[1]. These calculations are necessary because there can be an asymmetry in estimation in benefit-cost analysis when consumers' and producers' surpluses are taken to be part of the benefits of public projects and when surpluses foregone- that is, when money is spent by the government rather than citizens – are ignored (Dunn, 1967).

However, the literature is unnecessarily complicated, and this paper, therefore, clarifies the concepts involved. For example, to measure MCT, excess burden has to be calculated. But that being the case, it is incumbent on

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[1] The variation is associated with both different pieces of empirical evidence and with the methodology of calculation. For example, Topham (1985) calculated MCT at 1.21; Fullerton (1991) calculated MCT at 1.07and at 1.12 following the approach of Ballard, Shoven and Whalley (1985).

analysts to take account of any interactions between the level of service provided and the taxed commodities. This paper relies on the seminal paper by Diamond and Mirrlees (1971) where they emphasize the importance of public-good provision in determining optimal tax rules. The MCT is derived using the consumer's indirect utility function, and the measure is generalized to many commodities and many taxes. The measure derived is similar to that put forward by Diamond and Mirrlees. On the application side, the tax-base and public goods interaction has been largely ignored in previous analysis.

Finally the paper uses the assumption that demand functions are homogenous of degree zero in prices and income to substitute out the interaction terms and find an expression for MCT in terms of identifiable parameters. Whether MCT is greater or less than one depends on a number of easily-measured behavioral parameters. The paper uses some well-known examples to illustrate this. It is not difficult to envision situations in which MCT is less than one, and this should caution analysts against too readily accepting the view that MCT is anything between 1.07 and 1.21.

2. OBJECTIVE

The objective of the paper is to derive a formula for the measurement of the cost of the marginal revenues from indirect taxes (MCT) in terms of estimable demand parameters and in the context of cost-benefit analysis. In addition, the paper aims to highlight certain areas of interest to researchers for further investigation. The paper, also, tries to compare calculations of MCT with previous studies.

3. PREVIOUS STUDIES

Diamond and Mirrlees (1971), among others, included the interactions between the public good (z which is usually referred to as the government budget or the cost of public provision) and the taxed good (x) in the shadow price of the public good and showed that in the case of complementarities between the public good and the taxed good, an increment in the supply of the public good led to an increase in consumption of the taxed good; the revenue to be raised per unit of the taxed good is reduced, and benefits are enhanced. And vice versa in the case of substitutability which increases the burden and reduces the benefits (Diamond and Mirrlees (1971), Stiglitz and dasgupta (1971), and Atkinson and Stern (1974)). Topham (1985) showed that the MCT could be calculated by either the Marshallian or the Hicksian measure of excess burden. The former is equal to:

$$MCT = \frac{1-\zeta}{1+\gamma} \tag{1}$$

Where

$$\zeta = \sum t \frac{\partial x}{\partial z}$$

$$\gamma = \frac{1}{x} \sum t \frac{\partial x}{\partial t}$$

And $\frac{\partial x}{\partial t}$ represent ordinary or Marshallian effects.

While the latter is given by:

$$MCT = \frac{1-\xi}{1+\rho} \tag{2}$$

Where

$$\xi = \sum t \frac{\partial \times}{\partial z}$$

$$\rho = \frac{1}{\times} \sum t \frac{\partial \times}{\partial t}$$

And $\frac{\partial \times}{\partial t}$ represent substitution or Hicksian effects only.

Since $\gamma \neq \rho$, it follows that $\xi \neq \zeta$. In other words, it is not possible, in aggregate, to have zero income effects. The case where z has no effect whatsoever on the consumer's utility $\frac{\partial u}{\partial z} = 0$, is of no significance; for if the public good to be provided had no effect on the consumer's welfare, it would not be supplied.

Although it is necessary to include in the analysis the effects of public services on private demands, the literature does not provide evidence on the magnitude of these effects. There is little applied research devoted to this area. Thus, later on in this paper, it is necessary to substitute out all the $\frac{\partial x}{\partial z}$ terms in order to be able to calculate MCT in terms of easily measured demand parameters.

The Marginal Cost of Indirect Tax Revenue (MCT)

In the literature mentioned above, the efficiency conditions for public production were derived assuming that the government optimizes the level of public production (z) with respect to the tax structure. Hence, the marginal cost of indirect tax revenue (MCT) was derived at the optimum. In what follows, however, the government is not assumed to be a maximizing agent, and the corresponding social cost of providing a given level of z is calculated.

Consider a one-consumer economy, as the generalization to many consumers is straightforward. In this case, the indirect utility function of the consumer is given by: v=v(q,z,I) where q=p+t is a vector of post-tax prices, I represents income, and z denotes the cost of the public good. Differentiate the indirect utility function with respect to the tax vector t, holding I and v constant, to get:

$$\sum_i \frac{\partial v}{\partial t_i} + \sum_i \frac{\partial v}{\partial z}\frac{\partial z}{\partial t_i} = 0$$

Divide through out by the marginal utility of income (σ); using Roy's identity $\frac{\partial v/\partial t_i}{\sigma} = x_i$ and since $\frac{\partial v/\partial z}{\sigma} = MRS$ (MRS short for marginal rate of substitution between z and x which represents the real cost of z in terms of the numeraire x); it follows that:

$$\sum_i x_i + MRS\sum_i \frac{\partial z}{\partial t_i} = 0 \qquad (3)$$

Now from the government budget constraint z = tx, find:

$$\frac{\partial z}{\partial t_i} = x_i + \sum_j t_j \frac{\partial x_j}{\partial t_i}$$

And since the demand for x is given by x=x(q,I,z) then $\frac{\partial x}{\partial q}$ can be decomposed as:

$$\frac{\partial x}{\partial q_i} = \frac{\partial x}{\partial q_i} + \frac{\partial x}{\partial z}\frac{\partial z}{\partial q_i}$$

And as dq=dt it follows that:

$$\frac{\partial z}{\partial t_i} = x_i + \sum_j t_j \frac{\partial x_j}{\partial t_i} + \sum_j t_j \frac{\partial x_j}{\partial z}\frac{\partial z}{\partial t_i}$$

Which upon rearranging yields:

$$\frac{\partial z}{\partial t_i} = \frac{x_i(1+\gamma_i)}{1-\xi} \qquad (4)$$

Where

$$\xi = \sum_j t_j \frac{\partial x_j}{\partial z}$$

$$\gamma_i = \frac{1}{x_i}\sum_j t_j \frac{\partial x_j}{\partial t_i}$$

Using (4), equation (3) can be written out as:

$$MRS\left[\sum_i \frac{x_i(1+\gamma_i}{(1-\xi}\right] = -\sum x_i$$

Divide both the numerator and the denominator by $\sum x_i$ and denote $\frac{x_i}{\sum x_i}$ by m_i, then the equation above can be rearranged as:

$$MRS = -\frac{1-\xi}{\sum m_i(1+\gamma_i)}$$

And since $\sum m_i$ =1, it follows that:

$$MRS = -\frac{1-\xi}{1+\sum m_i\gamma_i} \qquad (5)$$

Equation (5) is similar to the formula derived by Wildasin (1979). It measures the MCT in general, regardless of the government's optimization behavior. It says that in order to keep the consumer's utility unchanged, the benefits from public provision should be equal to the term on the right-hand side, which represents the effects of the public good supply z on private demands x, and the sum of the changes in private demands in response to the tax changes.

If the tax structure is optimal (Sandmo (1976), among others); $\gamma_i = \gamma_j$, and (5) reduces to:

$$MRS = \frac{1-\xi}{1+\gamma} \tag{6}$$

Clearly (6) coincides with (1). It remains to apply the preceding analysis to actual data to get the magnitude of MCT or MRS. In the following section, it is necessary to use some assumptions regarding the specification of the demand function for the taxed good when the provision of the public good is asserted to affect the demand of the former.

The Magnitude of MCT

A strong assumption regarding the specification of the demand of the taxed good when the public good features as an argument in the function is to assume that the demand function is homogenous of degree zero. In other words, we assume that doubling all the arguments in the demand function does not affect demand[2]. This assumption can be relaxed but it complicates the mathematical arguments unnecessarily.

Given that governments provide services in order to increase welfare, (z) appears in the consumer's utility function and hence in the demand function for commodities. This creates an estimation requirement to either calculate the $\frac{\partial x}{\partial z}$ term or substitute it out. The latter approach, based on the supposition of rational behavior and thus with demand functions homogeneous of degree zero, yields estimable demand parameters.

The demand function for x can be expressed as: x = x(q,z,I), and from Euler's theorem it follows that:

$$0 = \sum_j \frac{\partial x_i}{\partial q_j} q_j + \frac{\partial x_i}{\partial z} z + \frac{\partial x_i}{\partial I} I$$

Multiply the equation above throughout by t_i to obtain:

$$0 = \sum_j \frac{t_i \partial x_i}{\partial q_j} q_j + \frac{t_i \partial x_i}{\partial z} z + \frac{t_i \partial x_i}{\partial I} I$$

Decompose the first-term on the right-hand side of the above equation using the Slutsky equation to get:

$$0 = \sum_j \frac{t_i \partial \times_i}{\partial q_j} q_j + \frac{t_i \partial x_i}{\partial z} z + \frac{t_i \partial x_i}{\partial I} I - \sum \frac{t_i \partial x_i}{\partial I} q_j x_j \tag{7}$$

Summation of (7) across all commodities yields:

$$\sum_i \frac{t_i \partial x_i}{\partial z} z = -\sum_i \sum_j \frac{t_i \partial \times_i}{\partial q_j} q_j \tag{8}$$

Using $\rho = \frac{1}{\times} \sum t \frac{\partial \times}{\partial t}$ equation (8) can be written as:

$$\sum_i \frac{t_i \partial x_i}{\partial z} z = -\sum_i \rho_i q_i \times_i \tag{9}$$

Rearrange (9), multiply the right-hand side by (I/I), and denote ($q_i x_i$/I) by (w_i) and (I/z) by (α), to obtain:

$$\xi = -\alpha \sum w\rho \tag{10}$$

Now substitute (10) into (5) to get:

$$MRS = \frac{1+\alpha \sum w\rho}{1+\sum m\gamma} \tag{11}$$

Decompose $\sum m\gamma$ using the Slutsky equation, and denote $\sum \frac{t\partial x}{\partial I}$ by ($\prod$) to get:

$$MRS = \frac{1+\alpha \sum w\rho}{1+\sum m\rho - \prod} \tag{12}$$

Equation (12) provides an expression for the MCT in terms of substitution and income effects of private demands. If the tax structure is optimal and thus ($\rho_i = \rho_j$) and ($\gamma_i = \gamma_j$), then (12) can be expressed in terms of elasticities as follows:

$$MRS = \frac{1+\alpha \sum r\varepsilon}{1+\sum r\varepsilon - \sum v\eta}$$

Where ε and η denote the compensated price and

[2] It is assumed here that doubling income and prices in the demand function (x=x(p,z,I) the cost of the public good is doubled and demand on the private good is unchanged.

income elasticities of x respectively, r denotes the tax ratio (t/q) and $v_i = \frac{t_i x_i}{I}$. And finally, by multiplying (v) with the expressions (I/I) and (z/z), and denoting (tx/z the tax share) by (s) we get:

$$MRS = \frac{1+\alpha\sum r\varepsilon}{1+\sum r\varepsilon - \frac{1}{\alpha}\sum s\eta} \qquad (13)$$

Equation (13) can be estimated in terms of broad commodity groups. The substitution terms can be evaluated using the Slutsky equation. Almost invariably the consensus in the literature is that MCT is larger than one; estimates range from 1.07 to 1.21, as we have observed. However, it is clear that once the substitutions for ($\frac{\partial x}{\partial z}$) are made, MCT may be more or less than one. Illustrations can be made by using data from household surveys but since such data is not available in Jordan yet, examples from British studies can be used so as to demonstrate comparisons with previous studies. Two-well known demand parameters from British studies can be used as an illustration.

In the case of owner-occupied houses, ε = -0.3, and η = 0.8. If it can be assumed that α = 2.5, and r = 0.2, then MCT = 1.12. In the case of cheese, by contrast, ε = -1.1, and η = 0.2. so that then MCT = 0.64. Combining the two commodities together, and letting the tax shares (the s_i) for owner-occupied houses and cheese equal, respectively, 0.8 and 0.2, we find (13) yields a value of MCT of 0.67. It is clear that MCT is less than one and this result is not a fanciful possibility, and these illustrations demonstrate the importance of including the tax-base and public-service interactions in the analysis of MCT.

4. CONCLUSION

The paper addressed the non-trivial issue of the measurement of the cost of marginal tax revenue (MCT).It demonstrated how the interactions between the public-good provided and the taxed commodities can be taken to account and be calculated in terms of easily-estimated behavioral parameters(i.e. income and price elasticities of demand). The paper used the requirement that demand functions are homogeneous of degree zero to substitute out the interaction terms and find an expression for the MCT in terms of easily-measured behavioral parameters. Using real-world data, its illustrations demonstrated that the conjecture that MCT could be less than one was not fanciful, and cost-benefit analysts, therefore, should be cautious before incorporating estimates of MCT grossly in excess of unity into their analysis.

In addition, the paper opens up new venues for researchers to incorporate better measurements of MCT by relaxing some of the assumptions in the paper and by adopting research along the lines of demand analysis so that the output of that research can be utilized by official decision makers when they devise public policy.

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تحليل تكاليف وأرباح الدخل الحدي من الضرائب غير المباشرة

*عامر باكير**

ملخـص

تتعرض الورقة باختصار الى الكتابات حول الأيرادات الضريبية الحدية وتستخلص معادلة لقياس المنفعة/التكلفة المترتبة عليها معتمدة على المعلمات القياسية لمؤشرات الطلب. وتستمد الورقة البعد النظري فيها بالاعتماد على ما قدمته ورقة الاقتصاديين ((Diamond and Mirrlees (1971) وغيرها من الأوراق العلمية المنشورة، وخاصة فيما يتعلق بالتداخل المتبادل بين السلعة العامة المنتجة من جهة والطلب على السلعة الخاضعة للضريبة من جهة أخرى. وتقوم الورقة باستبدال التداخل المتبادل هذا بما يعادله باستخدام نظرية (Euler) ومعادلة (Slutsky) وبافتراض ان معادلة الطلب متجانسة من الدرجة صفر.

وتحاول الورقة توضيح مفهوم المنفعة/التكلفة للأيرادات الضريبية الحدية وطريقة حسابها باستخدام البيانات الاحصائية المنشورة في الدراسات البريطانية وتقارن النتائج المحسوبة مع النتائج التي توصلت اليها الدراسات السابقة.

وتستنتج الورقة ان نسبة المنفعة الى التكلفة المترتبة على الأيرادات الضريبية الحدية قد تزيد عن الواحد صحيح وان الباحثين يجب ان يحتاطوا عند افتراضهم تقديرات تقل عن الواحد صحيح لهذه النسبة. وتبرز الورقة ايضا, بعض النواحي ذات الاهتمام للباحثين لعمل مزيد من الدراسات حولها.

الكلمات الدالة: التكلفة الحدّية، الضرائب غير المباشرة، السلعة العامة.

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