Modeling the Import and Export System of the United States: Using the Error Correction Model Representation

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ABSTRACT

This paper deals with the analysis of integration and cointegration of all seasonal frequencies for the export and import system of the United States in aggregated quarterly data that cover the period 1980-2003 (96 quarters). The used procedures are HEGY for the integration and EGHJ for the cointegration. The two time series are integrated at order 1 for all frequencies: 41, 21, 0 and 43. The commercial balance series is integrated at order 1 of the frequencies 0 and 21 and consequently it becomes stationary while filtering it by \((1 - B)^{(1 + B^2)}\). The cointegration analysis shows that these two time series are not cointegrated of the frequencies 0 and 21 whereas they are cointegrated of the frequency 41 (and ¾). Two cointegrating relations have been considered. The first relation (titled \(CR_{1,t}\)) has a constant deterministic part. The second relation (titled \(CR_{3,t}\)) has a deterministic part constituted of constant and three seasonal indicatory variables. For each of these two relations, we estimated the Error Correction Model (ECM) representation and we did some forecastings for the period 2002-2003 (8 quarters). The quality of the forecastings has been measured using the MAPE (Mean Absolute Percentage Error) criterion. The Chow test reveals a little structural change (at level 5 %) in the value of the parameters of the ECM representation using the cointegration relation \(CR_{3,t}\). But if we choose a level of 1% then we accept the null hypothesis that shows a non-structural change in the model.

Keywords: Export and import, United States, Cointegration, Error correction model, Structural change, Forecast.

1. INTRODUCTION

The occupation of the American United States by its foreign trade began seriously since the beginning of the 19th century. Arrangements have been made for the preparation of statistical accounts of trades of the United States with foreign countries to show the kinds, quantities and value of all exported and imported articles from each foreign country. After World War II, the United States took politics that encouraged its outside engagement. Indeed, this war has succeeded to a disorganization of the international economy, it ruined and ravaged a good part of Europe and Asia. The power of the United States became caricatural: half of the world industrial production and half of the international exchanges assured by only one country (See Le Diascorn and Blondel, 1995). This exceptional situation of the United States allowed the world to leave finally from the rut. This new reorganization of the "economy-world" imposed by the United States, was in conformity with the time to its interests to short and long-term and to the interests of the countries that adopt themselves the market economy. The monetary stability and the liberation of the exchanges of goods and funds were its liberal philosophy and free-trader. The efficiency of the American economy appeared then by a growth rates of GDP (the annual average) around 4% in 1960 accompanied by a receding more and more important until the time it reached the 0.9% in 1990, passed to (-0.5%) in 1991 (See Paul Kennedy, 1993: 366, 372). The industry of motor vehicles, consumer electronic products, engines and machinery, textiles, steel industry and computer products, a deterioration from the years 1970 (as exception, the aeronautics and chemical industry testified a remarkable expansion). Emmanuel Todd (2003) uses the deficit in the commercial balance of the United States as one of the indicators that predict the decomposition of the American system: "The

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savings rate of an American household is close to zero and every time that the American economy blooms, the imports increase and the deficit in the commercial balance increases also". That reveals a crisis in the American economy attested by the widening of the ditch between the exports and the imports (evolutive deficit of the commercial balance). Even on stock plan, in their analysis of the Dow Jones evolution since 1920, Frost and Prechter (1985) predict in 1990 the arrival of the so-called the overwhelming wave "C" that reaches its summit at the end of the year 2005 (An enormous fall in the Dow Jones value is expected)! (Using the Elliott Wave Approach) A lot of the publications treated the exports and imports of the United States like a couple in a larger macroeconomic system. For example: Clarida (1994) elaborated a model (in U.S. quarterly data) that reveals a unique cointegration relation between the consumer goods imports, the price of imports and the consumption of domestically produced varieties. Carone (1996) used the cointegration techniques in order to estimate the long-run equilibrium relationship between the aggregate demand for both total and non-oil merchandise imports of the United States. Oskooee and Brooks (1999) used Johansen and Juselius' cointegration procedure to test for the existence of a long-run relationship among the variables of import and export demand as well as estimates of the Marshall-Lerner condition. The import and export demand functions are estimated on a bilateral basis between the United States and each of its big trading partners considering a relation between the U.S. real import from trading a partner, with the U.S. real GDP and the real bilateral exchange rate between U.S. and trading partner. Senhadji and Montenegro (1999) studied the income and price elasticities of the export demand function for a lot of industrial and developing countries (included the United-States).

Our aim in this paper is to study the export and import system of the United States regardless of the other macroeconomic variables. We are going first to test for this system the presence of a common tendency, then to search for the real link between the exports and the imports. Since our data are quarterly, we will wait to find a seasonal integration for all frequencies (0, 1/4, 1/2 and 3/4). The research of a cointegration relationship for every frequency is to find a target that binds the two variables at long-run. In such a case, the fluctuations of short-term will be directed toward the long-term equilibrium. It is the objective of the Error Correction Model (ECM). In the case where the two variables will be cointegrated for a given frequency, their common evolution of long-term permits to put under control the export and import system of the United States. We signal that if the two variable exports and imports of the United States have a parallel evolution (a cointegration relation exists between them) then the system can reflect a crisis when a big ditch exists between them. But if the system evolves following a long-term target then it allows the authorities concerned to take into account this relation to intervene favorably for the interest of the country.

In economics, a lot of time series are available under aggregated data. The monthly data turn into quarterly or yearly data to have more information concerning the economical conjuncture of a certain country. This conjuncture is an important subject for the politicians. We study these time series in order to investigate their evolution and use the results in future planning. However, a lot of macroeconomic and financial time series are not stationary. The detection of the nature of the non-stationarity (i.e. whether the mean evolves as a polynomial of order 1 in time or the variance depends on the time) is important for all subsequent analysis because it permits us to specify the components of the adequate model and to obtain good forecastings. Thus, the abusive application of the filtering methods to make variables stationary conceals the properties of long-run equilibrium. In practice, most macroeconomic time series are dominated by an important seasonality. The specification of the nature of this seasonality (deterministic or stochastic) became primary in the work of the practitioners since a bad identification of the reason of the seasonality leads to a bad identification of the model and consequently we get unrealistic forecastings. Hylleberg et al. (1990) developed the HEGY procedure to test the seasonal unit roots in the case of the quarterly data. When the time series has a stochastic seasonal factor the ARIMA model is can not applicable. The advantage of the HEGY procedure, compared to that proposed by Dickey, Hasa and Fuller (DHF, 1984), is that it treats the seasonal integration frequency by frequency while the DHF procedure treats the seasonal filter as a whole (it accepts or refuses the seasonal integration of all frequencies). That is, the unit module counts for the DHF procedure and not the unit roots associated to a particular seasonal frequency. A lot of publications have been made using the HEGY.

The importance of the seasonal integration analysis consists in its use as a preliminary stage towards the cointegration. Engle and Granger (1987) demonstrated that the cointegrated time series can be represented by an ECM representation the textbook by Hamilton (1994) and Enders (2003) are good references on the subject. Of these, Hamilton is the more theoretical and Enders the more applied. The particularity of the ECM representation is to separate the static aspect of the model which is interpreted as a long – run equilibrium (the target), from the dynamic aspect (supposed to take into account different factors. The dynamic aspect is viewd a summary of the adjustments around this equilibrium (Gourieroux and Monfort, 1995, chapter 11). This ECM representation is both a static and dynamic model (For ECM representation, see Arino and Franses, 2000).

A lot of publications treated the mechanism of the cointegration of the frequency 0 that consists of searching a stationary linear combination between the I(1) time series (Johansen 1988; Johansen and Juselius, 1990) Perron (1989, 1997), Perron and Vogelslang (1992), Zivot and Andrew (1992), studied the cointegration of the frequency 0 by introducing an intervention variable that takes into account the change of the structure in a macroeconomic variable in Dickey-Fuller regression. Granger and siklos (1995) studied the effect of the aggregation on the cointegration of the frequency 0. However, it is very important to see if the cointegration can be accepted for all seasonal frequencies. A general procedure of the cointegration and seasonal cointegration has been developed by Lee(1992), and a procedure of two stages proposed by Engle and Granger (1987), Engle and Yoo (1987) for the frequencies 0 and ½. To test if there is a cointegration of the frequencies 0 or ½, it is necessary to test the stationarity of the linear combination between the integrated variables for these frequencies. To test this hypothesis, we achieve a Dickey - Fuller equation (DF) or an Augmented Dickey- Fuller equation (ADF). The ADF procedure tests the null hypothesis of non-stationarity, that is, $H_0$ (there is a unit root) against the alternative hypothesis $H_a$ (stationarity). The critical values are not the classic DF and ADF values because it is the residuals that are tested (Engle and Yoo, 1987; Mackinnon, 1991). Hylleberg et al (1993) proposed a cointegrating procedure of the frequencies $\frac{1}{4}$ and $\frac{3}{4}$. This procedure will be applied in our research.

The paper is organized as follows: First, we apply the HEGY procedure to analyze the integration for the seasonal frequencies of the two following aggregated time series in quarterly data: Export and import of the United States that cover the period 1980:1 - 2003:4 (96 quarters). Second, we use the EGHL procedure to study whether there is a cointegration between our two variables associated to the four seasonal frequencies $0, \frac{1}{4}, \frac{3}{4}$ and $\frac{1}{2}$. In the case where we get some cointegrating relations to a given frequency, we are going to estimate the corresponding ECM representation and carry out structural change testing and forecasting.

2. DATA

We have two macroeconomic variables (the export and import of the United States). The two time series (in millions of dollars) are aggregated in quarterly data and covering the period 1980-2003 (96 quarters). Each variable contains the following components: foods, feeds beverages, industrial supplies, consumer goods, capital goods, vehicles and other goods. The data are not corrected of the seasonal variations.

The inspection of the diagram below shows that, in the beginning of the period, the two variables are very close to each other, but during the last three years, an increasing ditch seems to wide them. Except the three first and last three years, the two variables reveal nearly a parallel behavior.

Now let us consider the diagrams of the variables filtered by the filters associated to different frequencies $0, \frac{1}{4}, \frac{3}{4}$ and $\frac{1}{2}$. We will denote the export (respectively the import) at time $t$ by $E_t$ (respectively $I_t$).

$E_t$ filtered by $(1 + B + B^2 + B^3)$: we observe an upward movement over time

$E_t$ filtered by $-(1 - B + B^2 - B^3)$: The variability increases over time

$E_t$ filtered by $(1 - B^3)$: We observe an important variability at the last years
$E_t$ filtered by $(1 - B^4)$: The obtained variable has a weak variability.

$I_t$ filtered by $(1+B+B^2+B^3)$

$I_t$ filtered by $-(1-B+B^2-B^3)$

$I_t$ filtered by $-(1 - B^2)$

$I_t$ filtered by $(1 - B^5)$

*The above diagrams have nearly a similar behavior to those associated to the variable of export.*
3. HEGY’S Method

The HEGY method is a general procedure for the unit root tests. It has been developed by Hylleberg, Engle, Granger and Yoo (1990). It consists in testing the presence of the module unit roots in the quarterly time series for all seasonal frequencies. The advantage of this method, compared to the other approaches proposed by Dickey-Fuller (1979, 1981), Dickey et al (1984) or Dickey et al (1986), is that it distinguishes between the different seasonal roots. Every root has a different interpretation because it designates a different cycle. It is not the module unit of a complex root that attracts the attention of the analysts of the time series but also the position of this root on the unit circle, that means, its seasonal frequency. In the case of the quarterly data, the seasonal filter \((1-B^4)\) is frequently used to make the series in question stationary. This filter is factorized as following:

\[
\Phi(B) = (\pi_1 B + \pi_2 B^2 + \pi_3 B^3 - \pi_4) (1 - B) (1 - B^2) (1 - B^3)
\]

\[
\Phi^*(B) = \pi_1 Y_{1,t} + \pi_2 Y_{2,t-1} + \pi_3 Y_{3,t-2} + \pi_4 Y_{4,t} + \mu_t + \epsilon_t
\]

where

\[
Y_{1,t} = (1 + B + B^2 + B^3) X_t, \quad Y_{2,t} = -(1 - B - B^2 - B^3) X_t, \\
Y_{3,t} = -(1 - B^2) X_t, \quad Y_{4,t} = (1 - B^4) X_t
\]

In the case where we accept the filter \((1-B^4)\) to make stationary \(X_t^*\), the \(Y_{k,t} \quad k = 1,2 \text{ et } 3\), have respectively, the unit roots of the frequencies 0, \(\pi\) and \(\pi^2\). For the roots 1, -1 and i, we consider the following null and alternative hypotheses:

\[
H_{01}: \Phi(1) = -\pi_1 = 0 \text{ (root 1)} \quad \ldots(5)
\]

\[
H_{02}: \Phi(1) > 0 \text{ or } \pi_1 < 0
\]

\[
H_{03}: \Phi(-1) = -\pi_2 = 0 \text{ (root -1)}
\]

\[
H_{04}: \Phi(-1) > 0 \text{ or } \pi_2 < 0
\]

\[
H_{05}: |\Phi(i)| = 0 \quad \pi_1 \cap \pi_3 = 0 \text{ (roots ± i)}
\]

\[
H_{06}: |\Phi(i)| > 0 \quad \sqrt{\pi_1^2 + \pi_3^2} \neq 0 \iff \pi_1 \neq 0 \text{ or } \pi_3 
eq 0
\]

If \(\Phi^*(B) = 1\) then the equation (3) becomes:

\[
Y_{1,t} = (1 + \pi_1) Y_{1,t-1} + \epsilon_t \quad \text{if} \quad \pi_2 = \pi_3 = \pi_4 = 0
\]

\[
Y_{2,t} = -(1 + \pi_2) Y_{2,t-1} + \epsilon_t \quad \text{if} \quad \pi_1 = \pi_3 = \pi_4 = 0
\]

\[
Y_{3,t} = -(1 + \pi_3) Y_{3,t-2} + \epsilon_t \quad \text{if} \quad \pi_1 = \pi_2 = \pi_4 = 0
\]

\[
Y_{4,t} = -(1 + \pi_4) Y_{4,t} + \epsilon_t
\]

The first and second equations will be treated as in the Dickey-fuller procedure of root unit testing, the third equation permits to test a stochastic seasonality in the case of biannual data. If the statistical results lead to the \(\pi_k \neq 0, k = 1,2,3,4\) then the \(X_t\) process is either stationary or it incorporates a deterministic seasonality and/or linear trend. If \(\pi_k \neq 0, k = 2,3,4 \text{ and } \pi_1 = 0\) then the \(X_t\) process requires a first difference to become stationary and the regression in (3) is equivalent to the standard test of ADF procedure. In general, if some \(\pi_k\) are zeros then the unit roots exist in the regression. In this situation, the HEGY procedure informs us that the corresponding \(Y_{k,t}\) are asymptotically not correlated because the corresponding unit roots have different frequencies. Finally, the HEGY procedure disposes the critical values of test statistics associated to the \(\pi_k\), \(k = 1,2,3,4\) and \(\pi_1 = \pi_3 \cap \pi_4\). These critical values vary according to the presence of a deterministic part in the regression or not. The optimal degree of the polynomial \(\Phi^*(B)\) results from an optimal specification by AIC using the auxiliary regression. The residuals of the corresponding model are accepted as a white noise when...
the Ljung – Box statistical is favorable. We pay more attention to the testing results when the seasonal indicatory variables are included in the regression. Actually, the inclusion of these variables in the equation, even if it is not necessary, don't provoke a deterioration in the power of the tests, on the other hand, an important bias results from their omission when they are necessary (See EGHL, 1993). The optimal orders of the AR models written under different deterministic parts are given in table (1) for the main and differentiated series. these orders vary between (1) and (5).

In the tables (2 and 3), we presented the statistics of unit roots of all frequencies. The inspection of the two tables shows that the export is integrated at order 1 of the frequencies 0 and 21, therefore, we can accept, the filter \((1 - B^2)\). The ¼ frequency seems masked, but the null hypothesis \((\pi_3 = \pi_4 = 0)\) seems accepted to the level 5% (seasonal auxiliary variables are included in the regression). For the Exports variable, we are going to consider the integration of all frequencies, therefore, the filter \((1 - B^4)\) is necessary to make it stationary. With regard to the import variable, the answers of the test statistics seem more lucid: the integration at order 1 is accepted of all frequencies, therefore, the stationarity of this variable is assured by the use of the filter \((1 - B)\times(1 + B^2)\) is necessary to make it stationary.

### Table 1

<table>
<thead>
<tr>
<th>Variables</th>
<th>Export</th>
<th>Δ(Export)</th>
<th>Import</th>
<th>Δ(Import)</th>
<th>(Import-Export)</th>
<th>IE</th>
<th>Δ(IE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Deterministic part a</td>
<td>p</td>
<td>AIC</td>
<td>p</td>
<td>AIC</td>
<td>p</td>
<td>AIC</td>
<td>p</td>
</tr>
<tr>
<td>--</td>
<td>5</td>
<td>16.71</td>
<td>4</td>
<td>16.70</td>
<td>5</td>
<td>17.17</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>5</td>
<td>16.72</td>
<td>4</td>
<td>16.68</td>
<td>5</td>
<td>17.21</td>
<td>4</td>
</tr>
<tr>
<td>C+SD</td>
<td>2</td>
<td>16.67</td>
<td>1</td>
<td>16.63</td>
<td>5</td>
<td>17.24</td>
<td>4</td>
</tr>
<tr>
<td>C+TR</td>
<td>5</td>
<td>16.67</td>
<td>4</td>
<td>16.72</td>
<td>5</td>
<td>17.24</td>
<td>4</td>
</tr>
<tr>
<td>C+TR+SD</td>
<td>2</td>
<td>16.61</td>
<td>1</td>
<td>16.67</td>
<td>5</td>
<td>17.26</td>
<td>4</td>
</tr>
</tbody>
</table>

a \(C = \) constant, \(TR = \) Trend, \(SD = \) seasonal indicatory variables: we have defined 3 auxiliary variables as follows:

\[
D_k = \begin{cases} 
1 & \text{if } t \text{ corresponds to the quarter } k, k = 1, 2, 3 \\
0 & \text{otherwise}
\end{cases}
\]

The quarter 4 has been considered as a basis.

The used general model is:

\[
\Phi^* (B) Y_{4t} = a + bt + c_1 D_{1t} + c_2 D_{2t} + c_3 D_{3t} + \pi_1 Y_{1,t-1} + \pi_2 Y_{2,t-1} + \pi_3 Y_{3,t-2} + \pi_4 Y_{3,t-1} + \varepsilon_t
\]

The optimal order \( p \) of \( \Phi^* (B) \) has been identified by the AIC criterion. The period of estimation is 1980-2001.

### Table 2

<table>
<thead>
<tr>
<th>Variables</th>
<th>Deterministic Parts</th>
<th>( \hat{\pi}_1 )</th>
<th>( \hat{\pi}_2 )</th>
<th>( \hat{\pi}_3 )</th>
<th>( \hat{\pi}_4 )</th>
<th>( \hat{\pi}_3 \cap \hat{\pi}_4 )</th>
<th>( Q_{12}^a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Export</td>
<td>-</td>
<td>-1.7</td>
<td>-0.09</td>
<td>-1.07</td>
<td>0.89</td>
<td>0.95</td>
<td>8.67</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>-2.4</td>
<td>-0.08</td>
<td>-1.11</td>
<td>0.86</td>
<td>0.98</td>
<td>6.66</td>
</tr>
<tr>
<td></td>
<td>C+SD</td>
<td>-4.04b</td>
<td>-1.60</td>
<td>-3.71b</td>
<td>1.04</td>
<td>7.76b</td>
<td>5.08</td>
</tr>
<tr>
<td></td>
<td>C+TR</td>
<td>-2.14</td>
<td>-0.08</td>
<td>-1.12</td>
<td>0.85</td>
<td>0.98</td>
<td>6.48</td>
</tr>
<tr>
<td></td>
<td>C+TR+SD</td>
<td>-4.01b</td>
<td>-1.62</td>
<td>-3.68b</td>
<td>1.11</td>
<td>7.73b</td>
<td>4.6</td>
</tr>
<tr>
<td>Import</td>
<td>-</td>
<td>-1.58</td>
<td>-0.4</td>
<td>-0.26</td>
<td>-0.16</td>
<td>0.05</td>
<td>10.4</td>
</tr>
<tr>
<td></td>
<td>C</td>
<td>-3.05b</td>
<td>-0.35</td>
<td>-0.33</td>
<td>-0.13</td>
<td>0.07</td>
<td>7.0</td>
</tr>
<tr>
<td></td>
<td>C+SD</td>
<td>-2.93</td>
<td>-1.1</td>
<td>-1.8</td>
<td>-0.05</td>
<td>1.62</td>
<td>6.9</td>
</tr>
<tr>
<td></td>
<td>C+TR</td>
<td>-3.87b</td>
<td>-0.33</td>
<td>-0.5</td>
<td>0.07</td>
<td>0.13</td>
<td>5.7</td>
</tr>
<tr>
<td></td>
<td>C+TR+SD</td>
<td>3.79b</td>
<td>-1.05</td>
<td>-1.95</td>
<td>0.18</td>
<td>1.92</td>
<td>5.3</td>
</tr>
</tbody>
</table>
The statistical \( Q_{12} \) follows a chi-squared of 12 degrees of freedom (the critical value is 21.03 at the level = 5%). Therefore the residuals associated to every type of deterministic parts are accepted as a white noise.

The test is significant at the level = 5% (The critical values are in HEGY 1990).

**Table 3**

<table>
<thead>
<tr>
<th>Variables filtered by ((1-B))</th>
<th>Deterministic parts</th>
<th>( \hat{\pi}_1 )</th>
<th>( \hat{\pi}_2 )</th>
<th>( \hat{\pi}_3 )</th>
<th>( \hat{\pi}_4 )</th>
<th>( \hat{\pi}_3 \cap \hat{\pi}_4 )</th>
<th>( Q_{12} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exports</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>-0.12</td>
<td>-0.12</td>
<td>1.33</td>
<td>0.89</td>
<td>9.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-0.12</td>
<td>-0.11</td>
<td>1.32</td>
<td>0.88</td>
<td>9.8</td>
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<td></td>
</tr>
<tr>
<td>C+SD</td>
<td>-1.43</td>
<td>-2.73</td>
<td>3.03</td>
<td>8.37 ( ^a )</td>
<td>13.5</td>
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<td></td>
</tr>
<tr>
<td>C+TR</td>
<td>-0.08</td>
<td>-2.66</td>
<td>1.30</td>
<td>0.86</td>
<td>10.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C+TR+SD</td>
<td>-1.41</td>
<td>-2.66</td>
<td>3.02</td>
<td>8.37 ( ^a )</td>
<td>14.2</td>
<td></td>
<td></td>
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<tr>
<td>Imports</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>-0.44</td>
<td>-0.26</td>
<td>0.04</td>
<td>0.04</td>
<td>11.7</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-0.44</td>
<td>-0.26</td>
<td>0.04</td>
<td>0.04</td>
<td>11.8</td>
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<td></td>
</tr>
<tr>
<td>C+SD</td>
<td>-1.29</td>
<td>-1.19</td>
<td>1.19</td>
<td>1.57</td>
<td>11.9</td>
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<tr>
<td>C+TR</td>
<td>-0.28</td>
<td>-0.08</td>
<td>0.00</td>
<td>0.04</td>
<td>11.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>C+TR+SD</td>
<td>-1.30</td>
<td>-1.19</td>
<td>1.13</td>
<td>1.51</td>
<td>11.6</td>
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<tr>
<td>IE(_r)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>-</td>
<td>-0.45</td>
<td>0.91</td>
<td>0.50</td>
<td>13.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>-0.45</td>
<td>0.89</td>
<td>0.48</td>
<td>13.3</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>C+SD</td>
<td>-1.60</td>
<td>2.32</td>
<td>3.88</td>
<td>8.06</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C+TR</td>
<td>-0.46</td>
<td>0.87</td>
<td>0.47</td>
<td>12.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C+TR+SD</td>
<td>-1.61</td>
<td>2.30</td>
<td>3.85</td>
<td>7.7</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( ^a \) The test is significant at the level = 5% (The critical values are in HEGY 1990).

\( ^b \) the Ljung-Box statistics (the critical value is 21.03 at the level = 5%).

### 4. SEASONAL COINTEGRATION

Let’s consider a vector \( X_t \) of dimension \( N \times 1 \) whose components have the frequencies 0, \( \frac{1}{4} \), \( \frac{1}{2} \), \( \frac{3}{4} \). The Wold representation permits to write \((1 - B^k)X_t = C(B)e_t\), where \( e_t \) is a white noise vector \( IND(0, \Omega) \) and \( C(B) \) is a \( N \times N \) polynomial matrix in \( B \). If there is a cointegration of the frequency 0 then there exists a \( N \times r_1 \) matrix \( A_1 \) \((N > r_1 \geq 0)\) such that \( A_1^t C(i) = 0 \), while a cointegration of the \( \frac{1}{2} \) frequency requires the existence of a \( N \times r_2 \) matrix \( A_2 \) \((N > r_2 \geq 0)\) such that \( A_2^t C(i) = 0 \). The columns of \( A_1 \) and \( A_2 \) are called the cointegrating vectors of the frequencies 0 and \( \frac{1}{2} \), respectively, while \( r_1 \) and \( r_2 \) are called the cointegrating ranks. The cointegration of the frequencies \( \frac{1}{4} \) and \( \frac{3}{4} \) that correspond to the complex roots i and -i, allows an extension of the notion of the cointegrating vector to the cointegrating polynomial \( A(B) = A_1 + A_2 B \) such that \( A_k C(i) = 0 \) \((k=3,4)\) where \( A_3 \) and \( A_4 \) are \( N \times r_3 \) matrices with \((N > r_3 \geq 0)\). In practice, instead of the Wold representation, we consider ECM representation suggested by HEGY (1990):

\[
A(B)Y_{4t} = \gamma_1 A_1^t Y_{1t-1} + \gamma_2 A_2^t Y_{2t-1} - (\gamma_3 + \gamma_4 B)(A_3 + A_4 B)Y_{3t-2} + e_t
\]

The left member of the equation is a VAR stationary process filtered by \((1 - B^k)\). The right member of the equation corresponds to the cointegrating relations of the different frequencies. \( \gamma_1, \gamma_2, \gamma_3 \) and \( \gamma_4 \) are matrices of dimensions \( N \times r_1, N \times r_2, N \times r_3 \) and \( N \times r_4 \) respectively. These matrices contain the weights of the
cointegrating relations of the different frequencies in the N equations. The polynomial \( A(B) \) has all the roots outside of the unit circle and \( A(0) = I_n \) (identity matrix). The cointegrating relations of the frequencies 0, \( \frac{1}{2} \), \( \frac{1}{4} \) and \( \frac{3}{4} \) are given respectively, by \( A_1Y_{1,t} \) (one cycle per year i.e. cointegrating relation at long term), \( A_2Y_{2,t} \) (two cycles per year) and \( (A_3^4 + A_4^1 B)Y_{3,t} \) (four cycles per year). Since our system is constituted of the two variables (import and export), it can exist at most one cointegrating relation of each frequency \( \theta \) \( (\theta = 0, \frac{1}{2}, \frac{1}{4} \) and \( \frac{3}{4} ) provided that each of our variables is \( I_1(\theta) \). If for each frequency the cointegrating rank is 1 then the ECM representation has the following form (See Bresson and Pitotte, 1995: 453-459):

\[
\begin{pmatrix}
A_{11}(B) & A_{12}(B) \\
A_{21}(B) & A_{22}(B)
\end{pmatrix}
\begin{pmatrix}
\Delta_t X_{1,t}^1 \\
\Delta_t X_{2,t}^2
\end{pmatrix}
= \begin{pmatrix}
g_{11} - g_{12}a_{12} \\
g_{21} - g_{22}a_{22}
\end{pmatrix}
\begin{pmatrix}
X_{1,t-1,1}^1 \\
X_{2,t-1,1}^2
\end{pmatrix}
+ \begin{pmatrix}
g_{21} + g_{24}B \\
g_{23} + g_{24}B
\end{pmatrix}
\begin{pmatrix}
X_{3,t-2}^1 - A_1X_{3,t-2}^2 - A_3X_{3,t-3}^2
\end{pmatrix}
+ \varepsilon_t
\]

Where

\[
X_{1,t}^k = (1 + B + B^2 + B^3)X_t^k,
\]

\[
X_{2,t}^k = -(1 - B^2 + B^3 - B^4)X_t^k,
\]

\[
X_{3,t}^k = -(1 - B^3)X_t^k,
\]

\( k = 1, 2 \) and \( X_{1,t}^1 \) (resp. \( X_{1,t}^2 \)) represents the export variable (resp. import variable). A cointegration associated to the frequency 0 is interpreted as an indication of a parallel movement at long-run between the non stationary time series. A cointegration of the other frequencies is also interpreted as a parallel movement in the corresponding seasonal. The \( X_{k,t}^1 \) and \( X_{k,t}^2 \) \( (k=1,2,3) \) have asymptotically an infinite variance (i.e. non stationary variable for a particular seasonal frequency) and all linear combinations will have an infinite variance except the linear combination \( I_6(0) \) for all value of \( \theta \):

\[
\begin{align*}
Z_{1t} &= X_{1,t-1}^1 - a_{12}X_{2,t}^2 \\
Z_{2t} &= X_{1,t-1}^1 - a_{22}X_{2,t}^2 \\
Z_{3t} &= X_{3,t-2}^1 - A_1X_{3,t-2}^2 - A_3X_{3,t-3}^2
\end{align*}
\]

The last linear combination is a dynamic relation.

### 4.1 Cointegrating test of the frequency 0:

We consider the two following regressions:

\[
X_{1,t}^1 = \phi X_{2,t-1}^2 + \eta_t
\]

\[
\Delta \eta_t = \pi_1u_{t-1} + \sum_{j=1}^{p} b_j \Delta u_{t-j} + \epsilon_t
\]

In the first equation, we can introduce a deterministic part constituted of only one constant or a constant and a linear trend. We can also consider the variable \( X_{1,t}^1 - X_{2,t}^1 \) and then apply the usual Dickey-Fuller test. The cointegrating strategy of the frequency 0 has been elaborated by Engle and Granger (1987), Engle and Yoo (1987). It is a procedure of two stages: the first stage leads to make a regression of \( X_{1,t}^1 \) on \( X_{2,t}^2 \). The resulting residuals are interpreted as the cointegrating linear relation. To this stage, the Durbin - Watson statistic DW is used to test the stationarity of the residuals. If the behavior of \( u_t \) reveals a unit root, then the DW will be close to 0 and consequently we cannot accept the hypothesis of the cointegrating relation. On the contrary, if the DW is significantly larger than 0, we accept the cointegrating relation of the frequency 0. The second stage consists in making an auxiliary regression that considers the second equation above to calculate the ADF statistic.

### 4.2 Cointegrating test of the frequency \( \frac{1}{2} \):

As in the previous case, we consider the two following regressions (See Engle et al., 1993):

\[
X_{2,t}^1 = \phi X_{2,t-1}^2 + \nu_t
\]

\[
(\nu_t + \nu_{t-1}) = \pi_2(\nu_{t-1}) + \sum_{j=1}^{p} b_j(\nu_{t-j} + \nu_{t-j-1}) + \epsilon_t
\]

The sign (-) is used so that the statistic distribution is
the same as $\Delta u_t$ on $u_{t-1}$, The cointegrating regression can be realized with or without the deterministic part.

4.3 Cointegrating test of the frequencies $\frac{\pi}{4}$ and $\frac{3\pi}{4}$:

We regress $X_{1,t}$ on $X_{2,t}$ and $X_{2,t-1}$. Hence we consider

$$X_{3,t} = \beta_1 X_{2,t}^2 + \beta_2 X_{3,t-1} + w_t,$$

$$(w_t + w_{t-2}) = \pi_3 (-w_{t-2}) + \pi_4 (-w_{t-1}) + \sum_{j=1}^p b_j (w_{t-j} + w_{t-j-2}) + e_t.$$

The null hypothesis of the cointegrating relation of the frequencies $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ implies that $\pi_3 = \pi_4 = 0$ in the auxiliary regression. The critical values of statistics associated to $\pi_3 = 0$, $\pi_4 = 0$ and $\pi_3 = \pi_4 = 0$ are presented by EGHL (1993). The comparison of the critical values of $\pi_3$ with those that are gotten by HEGY (1990), reveals that working on the estimated residues leads to a distribution that has bigger values. The inclusion of a constant in the cointegrating regression doesn't affect the distributions which are affected only by the inclusion of seasonal variables. In the following, we present the statistics for the different cointegrating tests (tables 4, 5, 6).

### Table 4: Cointegrating test of the frequency $0$

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variable $X_{2,t}$</th>
<th>Deterministic part</th>
<th>$R^2$</th>
<th>DW</th>
<th>Deterministic part</th>
<th>p</th>
<th>$Q_{n,c}^a$</th>
<th>ADF $t_{\pi_1}^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{1,t}$</td>
<td>0.727</td>
<td>-</td>
<td>0.94</td>
<td>0.02</td>
<td>-</td>
<td>1</td>
<td>$Q_{20} = 26$</td>
<td>-2.61</td>
</tr>
<tr>
<td></td>
<td>(88.8)$^d$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,t}$</td>
<td>0.667</td>
<td>C</td>
<td>0.95</td>
<td>0.02</td>
<td>-</td>
<td>1</td>
<td>$Q_{20} = 24.9$</td>
<td>-2.36</td>
</tr>
<tr>
<td></td>
<td>(38.9)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,t}$</td>
<td>0.361</td>
<td>C+TR</td>
<td>0.96</td>
<td>0.03</td>
<td>-</td>
<td>1</td>
<td>$Q_{20} = 26$</td>
<td>-2.62</td>
</tr>
<tr>
<td></td>
<td>(7.14)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,t}$</td>
<td>1 fixed</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>C+TR</td>
<td>5</td>
<td>$Q_{21} = 14.8$</td>
<td>-2.7</td>
</tr>
<tr>
<td>$X_{1,t}$</td>
<td>1 fixed</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>C+TR+SD</td>
<td>5</td>
<td>$Q_{21} = 17$</td>
<td>-2.66</td>
</tr>
</tbody>
</table>

*a* The critical values are given by Engle - Yoo (1987) and Dickey - Fuller (1979). For a sample of 100 observations and at the level of 5%, the critical values are -3.17 and -3.45 respectively. Since $t_{\pi_1} > t_{lab}$, we accept a unit root in the residues.

*b* The optimal order $p$ in the auxiliary regression is identified by the AIC criterion.

*c* $Q$ represents the Ljung – Box statistic. It permits to validate the empiric residues in the auxiliary regression as a white noise.

*d* In parentheses, we present the t ratios.

### Table 5: Cointegrating test of the frequency $\frac{\pi}{2}$

<table>
<thead>
<tr>
<th>Dependent variable</th>
<th>Independent variable $X_{2,t}$</th>
<th>Deterministic part</th>
<th>$R^2$</th>
<th>DW</th>
<th>Deterministic part</th>
<th>p</th>
<th>$Q_{n,c}^a$</th>
<th>ADF $t_{\pi_1}^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{1,2,t}$</td>
<td>0.761</td>
<td>-</td>
<td>0.61</td>
<td>2.56$^b$</td>
<td>-</td>
<td>1</td>
<td>$Q_{20} = 20.9$</td>
<td>-1.79$^a$</td>
</tr>
<tr>
<td></td>
<td>(12.2)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,2,t}$</td>
<td>0.81</td>
<td>C</td>
<td>0.61</td>
<td>2.45$^b$</td>
<td>-</td>
<td>1</td>
<td>$Q_{20} = 20.6$</td>
<td>-1.97$^a$</td>
</tr>
<tr>
<td></td>
<td>(11.6)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,2,t}$</td>
<td>0.565</td>
<td>C+SD</td>
<td>0.80</td>
<td>2.08$^c$</td>
<td>-</td>
<td>1</td>
<td>$Q_{20} = 18.3$</td>
<td>-2.6$^c$</td>
</tr>
<tr>
<td></td>
<td>(9.85)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*a* Since $t_{\pi_1} > t_{lab}$, we don't accept the cointegrating relation of the biannual frequency.

*b* The Durbin-Watson statistic leads to accept a correlated residues in the cointegrating regression.

*c* We don't accept the correlation in the residues.
Table 6: Cointegrating test of the frequencies $\lambda_4$ and $\lambda_3$

<table>
<thead>
<tr>
<th>Cointegrating regression</th>
<th>Auxiliary regression*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dependent variable</td>
<td>$X^1_{3,t}$, $X^2_{3,t-1}$</td>
</tr>
<tr>
<td>$X^1_{3,t}$</td>
<td>-0.008 (-0.13)</td>
</tr>
<tr>
<td>$X^1_{3,t}$</td>
<td>-0.017 (-0.26)</td>
</tr>
<tr>
<td>$X^1_{3,t}$</td>
<td>0.181 (2.83)</td>
</tr>
</tbody>
</table>

* For a sample size 100, the critical values at the level 5% are respectively 7.10 and 10.12 (cointegrating regression contains a constant C and C + SD) and 7.21 (cointegrating regression without deterministic part). Therefore we accept the cointegrating relation of the frequencies $\lambda_4$ and $\lambda_3$.

** we accept $\pi_4 = 0$ and we reject $\pi_3 = 0$.

c The corresponding values of AIC are respectively 16.54, 16.14 and 16.55.

4.4 Cointegrating Relations

The inspection of the results presented in the table (4) shows that the ADF statistics, in all cases, indicate the presence of a unit root and by consequence there isn't a long-term cointegration. We conclude that the filter (1-B) is not adequate. In the same way, the table (5) shows that there is not a cointegration of the biannual frequency. Finally, the results of the tests in the table (6) leads to the acceptance of the hypothesis cointegration of the frequencies $\lambda_4$ and $\lambda_3$. This cointegration reveals a parallel movement between the two non stationary series i.e. it indicates that the yearly seasonal behaviors of the export and the import of the United States are similar.

Since we accepted the cointegration of the frequencies $\lambda_4$ and $\lambda_3$ (table 6), the cointegrating regression (dynamic specification) is considered first without deterministic part, then with deterministic part which contains either only a constant alone, or a constant and the seasonal auxiliary variables $D_{kt}, k = 1,2,3$). Therefore, we got a cointegrating relation between $X^1_{3,t}$ and $X^2_{3,t-1}$ under three different shapes:

$$\text{RC}_{1,t} = X^1_{3,t} + 0.008X^2_{3,t} - 0.475X^2_{3,t-1} \quad (0.13) \quad (-7.46)$$

$$\text{RC}_{2,t} = X^1_{3,t} + 401.03 + 0.017X^2_{3,t} - 0.464X^2_{3,t-1} \quad (0.61) \quad (0.26) \quad (-7.04)$$

$$X^1_{3,t} = 1.2055 + 6597D_{1t} + 4343D_{2t} \quad (-2.0) \quad (4.8) \quad (3.1)$$

$$- 2583D_{3t} - 0.181X^2_{3,t} - 0.345X^2_{3,t-1} \quad (-1.8) \quad (-2.8) \quad (-5.3)$$

By analogy to the ECM representation of the non seasonal case, we consider the ECM version proposed by Osborn (1993), we write:

$$\Delta_4 E_t = \sum_{i=1}^{p} a_1 \Delta_4 E_{t-i} + \sum_{j=1}^{q} b_j \Delta_4 I_{t-j} - (b_{11} + b_{12}B)\text{RC}_{1,t} + \epsilon_{1t}$$

$$\Delta_4 I_t = \sum_{i=1}^{p} a_1 \Delta I_{t-i} + \sum_{j=1}^{q} b_j \Delta_4 E_{t-j} - (b_{21} + b_{22}B)\text{RC}_{1,t} + \epsilon_{2t}$$

with $E_t$ and $I_t$ defined as before,

$$E_{1,t} = -(1 - B^2)E_t \quad \text{and} \quad I_{1,t} = -(1 - B^2)I_t \quad.$$ The

$$\text{RC}_{i,t} \quad i = 1, 2, 3, \text{ is one of the above cointegrating relations. The estimation of the ECM representation is the}$$
Cointegrating relation without deterministic part:

\[
\Delta_4 E_t = 0.877 \Delta_4 E_{t-1} - 0.485 \Delta_4 E_{t-4} + 0.382 \Delta_4 E_{t-5} + 0.24 \Delta_4 I_{t-1} - 0.164 \Delta_4 I_{t-3} - 0.259 RC_{1,t-3} + \hat{\varepsilon}_{1t} \\
(9.95) (-3.62) (3.48) (4.57) (-3.21) (-2.64) (3332)
\]

\[
\Delta_4 I_t = 1.414 \Delta_4 I_{t-1} - 0.256 \Delta_4 I_{t-2} - 0.871 \Delta_4 I_{t-4} + 0.648 \Delta_4 I_{t-5} - (0.276 - 0.266 \beta) RC_{1,t-2} + \hat{\varepsilon}_{2t} \\
(14.46) (-1.97) (-6.51) (5.53) (2.29) (-2.18) (4854)
\]

Cointegrating relation with constant and seasonal indicatory variables:

\[
\Delta_4 E_t = 0.873 \Delta_4 E_{t-1} - 0.477 \Delta_4 E_{t-2} + 0.275 \Delta_4 I_{t-1} - 0.849 RC_{3,t-2} + \hat{\varepsilon}_{1t} \\
(7.9) (-5.8) (5.9) (-6.6) (3392)
\]

\[
\Delta_4 I_t = 1.221 \Delta_4 I_{t-1} - 0.951 \Delta_4 I_{t-4} + 0.571 \Delta_4 I_{t-5} + 0.125 \Delta_4 I_{t-9} - (0.261 - 0.249 \beta) RC_{3,t-2} + \hat{\varepsilon}_{2t} \\
(26.03) (-7.17) (4.37) (1.93) (1.57) (-1.45) (4920)
\]

5. Choice of the cointegrating relation

To choose between the two above cointegrating relations, we evaluated the forecasts for the period 1999-2003 (20 quarters) using the corresponding ECM. Indeed, the calculation of the forecasts was always a main object for the economic series. Forecasting and especially forecasting with the minimum error requires the following:

1. Knowing the reason of the non-stationarity of the economic time series
2. Detecting the nature of seasonality: deterministic or stochastic
3. Distinguishing between seasonal integration of certain frequency and all seasonal frequencies, etc.

Frances (1991) achieved a comparison of the forecasting performance between the models that incorporate the multiplicative Box-Jenkins filter \((1 - B)(1 - B^4)\) and those that require the filter \((1 - B)\) accompanied by a constant and 11 seasonal indicatory variables (FDSDm model). The result of this comparison revealed that the absence of the seasonal unit roots can have important implications in the forecasts. In the following, we are going to present some briefly convenient techniques that help to evaluate forecasts by the proposed models.

Let's designate by \(P_t\) the forecasted value and \(A_t\) the real value of a time series. If \(P_t = A_t\) then the forecasts are perfectly exact and the linear correlation coefficient between \(P_t\) and \(A_t\) is equal to 1 (\(t = 1, 2, \ldots, H; H = \) sample size of forecasts). However, this case is unrealistic because in all modelling, there are errors due to the uncertain factors not explained by the proposed model, and other errors due to a fast decision when the statistical test behaviors are impertinent. To examine the accuracy of the forecasts, we are going to analyze the behavior of the \(P_t\) and \(A_t\) sequences using the measures most used by the analysts of forecasts. From these measures, we mention:

* The order optimal \(p\) is identified by the AIC criterion: we first identified \(p\) considering the AR(\(p\)) model for \(\Delta_4 E_t\) in the cointegrating relation. We found AIC(5) = 16.62, then we introduced the variable \(\Delta_4 I_t\) and we found AIC(3) = 16.52. Similarly for the variable \(\Delta_4 I_t\), we found AIC(5) = 17.16 then AIC(1) = 17.2.

** In the cointegrating relation with constant, we note that this constant is not significant, therefore the ECM representation associated is practically the same that has been estimated in the first case (without deterministic part). For the third cointegrating relation, the order optimal \(p\) has also been determined by the AIC criterion (AIC(2) = 16.6 then AIC(1) = 16.4 for the first equation of the ECM ) and AIC(5)=17.2 then AIC(1) = 17.26 for the second equation.
1- Root Mean Squared Error:

\[ \text{RMSE} = \sqrt{\frac{1}{H} \sum_{i=1}^{H} (A_i - P_i)^2} \]

From this criterion, we can calculate the different sources of the forecast errors (For application of these techniques, see Joutz – Stekler, 2000). To get the different components of MSE, we follow the next one:

\[
\text{MSE} = \frac{1}{H} \sum_{i=1}^{H} (A_i - P_i)^2 = \frac{1}{H} \sum_{i=1}^{H} [2 \frac{1}{H} \sum_{i=1}^{H} (A_i - \bar{A})^2 + \frac{1}{H} \sum_{i=1}^{H} (P_i - \bar{P})^2 + (\bar{A} - \bar{P})^2 - \frac{2}{H} \sum_{i=1}^{H} (A_i - \bar{A})(P_i - \bar{P})] = (\bar{A} - \bar{P})^2 + \sum_{i=1}^{H} (A_i - \bar{A})^2 + \sum_{i=1}^{H} (P_i - \bar{P})^2 + \left(1 - r^2\right) \sum_{i=1}^{H} S_p^2
\]

where \( \bar{P} = \frac{1}{H} \sum_{i=1}^{H} P_i \), \( \bar{A} = \frac{1}{H} \sum_{i=1}^{H} A_i \), \( S_A \) and \( S_P \) are respectively the standard deviations of the sequences \( A_i \) and \( P_i \), and \( r \) is their linear correlation coefficient. Dividing the two members by MSE, we get

\[ 1 = U^M + U^R + U^D \]

\( U^M = \frac{(\bar{A} - \bar{P})^2}{\text{MSE}} \) measures the source of the forecast error resulting from an under-prediction or over-prediction of the average \( \bar{P} \)

\[ U^R = \frac{(S_A - r S_P)^2}{\text{MSE}} \]

represents the source of the forecast error resulting from an under-prediction or over-prediction of the regression slope of \( A_i \) on \( P_i \), \( t = 1, \ldots, H \).

\[ U^D = \frac{\left(1 - r^2\right) S_p^2}{\text{MSE}} \]

is the random source of forecasting.

In practice, we wish that \( U^D \) is close to 1 and \( U^M \) and \( U^R \) are near 0. The information about the quality of forecasts obtained by \( U^M \), \( U^R \) and \( U^D \), is very important particularly if \( H \) is large (\( H \geq 30 \)).

2- Mean Absolute Percentage Error:

\[ \text{MAPE} = \frac{1}{H} \sum_{i=1}^{H} \frac{|A_i - P_i|}{A_i} \times 100 \]

With \( \text{APE}_i = \frac{|A_i - P_i|}{A_i} \) (Absolute Percentage error).

This criterion is the most advisable to choose among two or several proposed models.

### Export variable

<table>
<thead>
<tr>
<th>Cointegrating relation RC1</th>
<th>Cointegrating relation RC2</th>
</tr>
</thead>
<tbody>
<tr>
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<tr>
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<td>( U^D )</td>
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### Import variable

<table>
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<tr>
<th>Cointegrating relation RC1</th>
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The inspection of the one-step ahead forecast errors shows clearly that for the two variable export and import, the relation $RC_1$ provides the best forecastings. For the horizons 2002-2003 (8 quarters), the statistical MAPE is 3.5% for the export variable and 4.6% for the import one. The real balance passed from ($\approx -468$) billions $^*$ dollars in 2002 to ($\approx -532$) billions of dollars in 2003. On the other hand, the predicted balance deduced from cointegrating relation $RC_3$ passed from ($\approx -487$) in 2002 to ($\approx -522$) in 2003. This widened the gap in the commercial balance can be considered as a sign of crisis in the American economy. In the literature of the cointegration, Perron (1989, 1994, 1997), Zivot and Andrews (1992), proposed to introduce in the Dickey-Fuller regression, an intervention variable that takes into account the change of the structure in a macroeconomic time series. Yet, this procedure doesn't take into account the seasonal integrations and its power is limited to test the integration of the frequency zero, in this case, the procedure chooses the ECM representation.

6. STRUCTURAL CHANGE TESTING

The stability of the parameters of a model plays an important role when we try to understand the economic mechanisms and to achieve some projections. Their stability can reflect some structural or punctual phenomena in the time. In order to study the temporal instability, the econometricians have, since some years, raised one of the fundamental hypotheses: the constancy of the coefficients in the time. The principle is to investigate the variability of the coefficients in the time. A lot of tests of stability of the parameters proposed since the year 1960 like the likelihood ratio test proposed by Quandt (1960), the Chow test proposed by Chow (1960), the Cumulative Sum of recursive residuals (Cusum test) and Cumulative Sum of Squares of recursive residuals (Cusum Square test) proposed by Brown, Durbin Evans and (1975, the test of influence of Belsey,Kuh and Welsh (1980). In the following, we limit ourselves to the Chow test for its easiness of setting in practice.

The null hypothesis means the absence of structural change. The evolutive Chow test is based on the following statistic:

$$Chow = \frac{SSR - (SSR_{1h} + SSR_{2h})}{(SSR_{1h} + SSR_{2h})} \times \frac{T - 2K}{K}$$

where SSR is the Residual Sum of Square of the model estimated on the whole T observations of the time series. The result can be regarded as obtained from a pooled sample comprising both the first $T_{1h}$ periods and the last $T_{2h}$ periods. $SSR_{1h}$ (resp. $SSR_{2h}$) represents Residual Sum of Square of the model estimated on $T_{1h}$ first observations(respectively on the $T_{2h}$ observations) with $T_{1h} = 36+4h$ and $T_{2h}=T-T_{1h}$, $h = 0,1,\ldots,5$. The choice of these two sample sub-periods has been made respecting the requirements of the Central Limit Theorem (sample size > 30). The statistic of Chow follows a Fisher distribution $F(K,T-2K)$. Using the cointegrating relation $RC_3$ in the ECM representation, we obtain the following plots:

The graphical representation of the statistics of Chow is located below of the statistic of Fisher at 1 % level of significance. It means, for this level, that there is no structural change in the value of the parameters for two equations. The inspection of the graphics at 5 % level shows that the export and import system of the United

* $1\text{ billion}=10^9$
States is provided with a certain little rupture: The biggest is associated to the sub-periods 1980:1-1993:4 and 1994:1-2003:4 for the export variable and to the sub-periods 1980:1-1989:4 and 1990:1-2003:4 for the import variable. The graphic aspect of two time series reveals that a tendency of divergent evolution appeared more and more important between the exports and the imports of the United States and by consequently there would be a possibility to lose the proposed cointegration relationship between them.

7. CONCLUSION:

The first point of our conclusion is that each of the two variables (Export and Import of the United States) is not stationary and each requires a specific treatment to become stationary. Indeed, the two series are integrated of all seasonal frequencies 0, ¼, ½, ¾ which means that they become stationary after the application of the filter \((1 - B^2)\). The balance commercial variable resulting from the difference between the import and the export is integrated of the frequencies 0, ¼, and ¾. This series becomes stationary after filtering by \(1 - B\times(1 + B^2)\). It encourages the idea that the biannual frequency has been eliminated.

The second point of the conclusion is related to the cointegration of the two series. The whole of our statistical results reinforces the idea of the non cointegration of the frequencies 0 and ½. on the other hand, the cointegration of the yearly frequency seems clear. This implies that outside a stochastic trend, the two series have a similar yearly seasonal behavior and by consequently they are dominated by a certain parallel yearly movement. This result is very close to that found by Engle et al. (1993) in the analysis of the seasonal cointegration concerning the consumption and the income in Japan.

The third point of the conclusion deals with the analysis of the forecasting performance of the ECM representation. Indeed, the stationary linear combination of the two series associated to the ¼ frequency can occur with a deterministic part (constant, or constant and indicatory seasonal variables) or without deterministic part in the cointegrating relation. The cointegrating relation RC3 (dynamic cointegrating relation) that results from a regression of \(X_{3,t}^1\) on \(X_{3,t}^1\) and \(X_{3,t}^1\) seems more adequate with the reality of the export and import system of the United States. The comparison of the quality of the forecastings between RC1 and RC3 permits to adopt the RC because it best reflects the reality of the system. The quantity RC3 is known as a disequilibrium error. It will of course, take a zero value when the export and import system is in equilibrium (long-run equilibrium) i.e:

\[
\Delta_2 E_t = DP_t + 0.181 \Delta_2 I_t + 0.345 \Delta_2 I_{t-1} \\
DP_t = -2055 + 6597 D_{1t} + 4343 D_{2t} - 2583 D_{3t} \\
\Delta_2 = (1 - B^2)
\]

DP, represents the deterministic part and \(D_{kt} k=1,2,3\), indicates the seasonal indicatory variables (the quarter 4 has been considered as a basis). The survey results of the stability in the time of the parameters show the fragility of this cointegration relation at 5 % level (but the stability of parameters is accepted at 1 % level). If the divergent evolution continues in the future (rise in the imports and decrease in the exports) then it would have an aggravation of the crisis in the American commercial balance. The loss of such a parallel evolution is going to a certain mess in the trade policy of the United States.

Finally, we suggest to apply this econometric models on the economics of the developing countries, and particularly the Arabian ones because this permits to a best planning.
### Data Anxexe

**Quarterly Data, 1980Q1 to 2003Q4 (96 observations)**

In millions of dollars

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<th>Imports</th>
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**Note:** Data sources: United States Department of Commerce, Economics and Statistics Administration, U.S. Census Bureau, Bureau of Economic Analysis.

### REFERENCES


Dickey, D.A. and Fuller, W.A. 1979. Distribution of the


تشكل نظام الاستيراد والتصدير في الولايات المتحدة

باستخدم تموذج الأخطاء المصححة

محمود نجيب مراد

ملخص

تهدف هذه الدراسة إلى تحليل التكامل والتكتل الفصلي المشترك لظاهرة الاستيراد والتصدير في الولايات المتحدة الأمريكية لكل من التوجهات الزمنية الأربعة، وذلك عبر دراسة البيانات الفصلية التي تغطي الفترة الزمنية من 1980 إلى 2003. لذا استخدمنا طريقة HEGY لدراسة التكامل، وطريقة EGHL لدراسة التكامل المشترك. وقد بينت نتائج الدراسة أن كلاً من الورادات والصادرات في أمريكا تحتوي على التكامل في الدرجة الواحدة لكل من التوجهات: ¼, ½, ¾, 0. أما بالنسبة لظاهرة الميزان التجاري، فإنها متكتلة ذات درجة واحدة مع التوجهات: 0, ½، وبالتالي تصبح ساكنة بعد تجميعها بواسطة ($1 - B$)($1 - B^2$)، وأظهرت النتائج أن الورادات والصادرات تمثلت كتكاملً مشتركً مع كل من التوجهات: ¼, ½, ¾، ولا تتكامل مع التوجهات: 0.½، وبعده خُصصنا إلى بناء علاقتين للتكامل المشترك: الأولى تحتوي على كمية ثابتة في القمة الحتمي، والثانية تحتوي على كمية ثابتة مع ثلاثة مغامرات تأنيط موسمية. وقد فحصنا تلك من خلال استخدام التحقق من تأنيط نموذج ديمون: 2003 نشط. وملاحظة من الأخطاء المصححة ودرسنا بعد ذلك جودة هذه التوقعات باستعمال معيار MAPE، وعندما لا يتواجد نسبة الخطأ في التوقعات 3.5% بالنسبة للصادرات و4.6% بالنسبة للورادات، وذلك باستخدام علاقة التكامل المشترك الثنائية (كمية ثابتة مع ثلاثة مغامرات تأنيط موسمية في القمة الحتمي). وأخيراً، فحصنا باختبار التغير الهيكلي لظام الصادرات والورادات الأمريكية وذلك باستخدام اختبار Chow الذي أظهر استقراره واضحًا، وبنى لحثنا بعض التغير الهيكلي بمتوسط معنوي 5%. الكتلات الدالة: الصادرات والورادات، الولايات المتحدة الأمريكية، التكامل المشترك، تموذج الأخطاء المصححة، التغير الهيكلي، التوقعات.

كلية العلوم الاقتصادية وإدارة الأعمال، الجامعة اللبنانية، النبطية، بيروت، لبنان. تاريخ استلام البحث 7/11/2005، وتأريخ