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The Welfare Loss of a Sales Tax in Jordan

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ABSTRACT

This paper shows how to measure the welfare loss of a 20% sales tax in Jordan for the year 2004. It tries to fill the gap between microeconomic theory and applied research in the area of welfare economics. Applied research in this area in Jordan seems to be lacking. Initially, this paper retrieves the analysis by Hausman (1981) where he derives the expenditure function from the ordinary demand function and uses it to measure the consumer surplus. This paper clarifies some ambiguity in that paper and corrects it utilizing the Stone-Geary utility function (usually referred to as the Linear Expenditure System (LES), ($u = (x_1+a)^{\alpha} (x_2+b)^{\beta}$)) and its corresponding demand functions to find a differential equation whose solution yields the substitution function which is used to measure the welfare loss of a 20% sales tax using 2004 data in Jordan.

The paper summarizes the theoretical issues involved, provides illustrations using the well known utility specifications (Stone-Geary and Cobb-Douglass), generalizes the analysis and applies it to Jordan using 2004 data.

The paper concludes that using ordinary demand functions the welfare loss of imposing a 20% sales tax in Jordan for the year 2004 amounted to JD 936 Million. The paper recommends that further investigation can be undertaken in this area by exploring the use of different demand specifications, differential taxes, price and income elasticities and different population groups.

Keywords: Measurement of Welfare Loss, Sales Tax, Expenditure Function, Substitution Function, Consumer Surplus.

1. INTRODUCTION

A library search of published applied microeconomic research on welfare effects of indirect taxes in Jordan in refereed Journals showed no results and such research seems not to exist. Accordingly, this paper tries to fill the gap between microeconomic theory and applied research in the area of welfare economics. Specifically, it tries to measure the welfare loss from indirect taxes. Initially, the paper relies on the analysis made by Hausman (1981), clarifies some ambiguity in his analysis, effects some corrections and expands his approach. Also, the paper uses the non linear form of ordinary demand functions to measure the welfare loss from imposing a 20% sales tax in Jordan for the year 2004. Finally, the paper concludes with results and recommendations.

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2. PROBLEM

There is a gap between microeconomic theory and its applications in Jordan, especially in the area of welfare economics. This gap arises from several reasons such as the non availability of data and the absence of practical methodologies. Specifically, it seems that there have been no research attempts that employ a microeconomic approach to measure the welfare loss from indirect taxes in Jordan.

3. OBJECTIVE

This paper shows how to measure the effect of a sales tax using nonlinear demand curves and utilizing the expenditure function route that was proposed by Hausman (1981). This paper clarifies a specific ambiguity in that paper and generalizes the analysis to taxes on several commodities. It applies the analysis using real data and suggests further research to be undertaken by scholars in this area.

4. PREVIOUS STUDIES

The research on applied welfare economics in Jordan in the area of welfare effects of indirect taxes is lacking. All the research attempts were undertaken using a macroeconomic approach. For instance, two relevant studies by Abu Hammour and Naser (1989), and Malkawi and Abu Hammour (1999) explored both Tax burden and tax effort for the economy of Jordan and tried to show whether the tax burden was below or exceeded the tax capacity. In those papers, a macroeconomic approach was adopted. Tax burden was measured as the ratio of tax revenues including fees and licenses to GNP. Other research on indirect taxes in Jordan was undertaken in the Jordanian universities as part of the Masters program. Although that work was not published yet the approach was at the macroeconomic level, relating indirect taxes to macroeconomic variables such as GDP, Consumption and Investment.

The literature on actual measurement of tax burdens is rather meager and none of similar work was undertaken in developing countries. I believe that this study is pioneering in Jordan.

5. THEORETICAL BACKGROUND

The theory of consumer behavior as outlined in this paper is abundantly available in the literature. The utility function dates back to Debreu, The indirect utility function is attributed to Roy and the expenditure function is associated with Hicks. The duality approach to consumer theory follows the work of McFadden and Winter. A thorough description of the underlying concepts and analysis can be found in most advanced microeconomic textbooks such as Varian (1992), Layard and Walters (1978), and Birchenhall and Grout (1984). Also, the use of demand functions to derive the indirect utility function is due to Hausman (1981). The literature on welfare effects of indirect taxes can be obtained by referring to Atkinson and Stiglitz (1980), Musgrave and Musgrave (1980), Diamond and McFadden (1974), Browning (1987) and Boadway (1979). Finally, the solution of differential equations can be found in Ayres (1952), Straud (1981), Chiang (1984), and Birchenhall and Grout (1984).

In 1981, Hausman attempted to derive the expenditure function from the Marshallian demand curve. He employed Roy's identity to find a differential equation which relates the change in expenditure required to keep the consumer's indirect utility constant in response to a price change. Subsequently, he integrated the differential equation to yield a relationship between expenditure and prices which he used to measure the consumer's surplus and the deadweight loss (DWL). In this paper, however, it is shown that Hausman's method conveys a misconception of interpretations. The solution of the differential equation and the expenditure function are not the same. The former relates to the ordinary (Marshallian) demand while the latter corresponds to the compensated (Hicksian) demand.

This paper assumes a LES utility function and the corresponding indirect utility and expenditure functions are derived and are used to calculate the consumer's surplus (CS). In contrast, Hausman's method is employed to calculate the CS. The results show no differences in the magnitudes of the CS for the two methods of measurement. Both methods yield the same result; however, there is some ambiguity in Hausman's analysis which is clarified. Also a more general analysis is presented.

5.1. Hausman (1981) Analysis

In this section, the analysis by Hausman for the LES utility function
($u=(x_1+a)^{\alpha}(x_2+b)^{\beta}$) is demonstrated. Maximization of the consumer's LES utility function yields the following forms for the consumer's indirect utility, expenditure and Marshallian demand functions (Varian, 1992; Layard and Walters, 1978; Birchenhall and Grout, 1984):

$$v(p,I)=\frac{(I-\mathrm{a}p_1-bp_2)^{\alpha+\beta}}{\mathrm{A}{p_1}^{\alpha}{p_2}^{\beta}} \qquad (1)$$

$$E(p,u)=(Avp_1^{\alpha}p_2^{\beta})^{\frac{1}{a+b}}+\mathrm{a}p_1+bp_2 \qquad (2)$$

$$x_1(p,I)=\frac{I+\frac{\beta}{\alpha}\mathrm{ap_1\text{-}bp_2}}{\frac{\alpha+\beta}{\alpha}p_1} \qquad (3)$$

Where $v(p,I)$ represents the consumer's indirect utility; $E(p,u)$ represents the consumer's expenditure function; p_1 and p_2 denote the prices of two commodities chosen arbitrarily; the parameters a and b represent the subsistence levels; α and β denote substititution elasticities; A is equal to
$((\alpha+\beta)^{\alpha+\beta})/(\alpha^{\alpha}\beta^{\beta})$.

5.2. Illustration

The purpose of this section is to clarify the ambiguity in Hausman's paper. The following equations are explained thoroughly by Birchenhall and Grout (1984).

To illustrate, assume the following values: p_1=20; p_2=10; I=2000; $\alpha = \beta$=0.5; a=b=1. Hence (3) becomes (4):

$$x_1(p_1, I) = \frac{I + p_1 - p_2}{2p_1} \tag{4}$$

From Roy's identity obtain:

$$\frac{\frac{\partial v}{\partial p_1}}{\frac{\partial v}{\partial I}} = -x_1(p, I) \tag{5}$$

Also, from the indirect utility function, the effect of changes in p_1 and I holding utility constant can be expressed as (Hausman 1981):

$$dv = 0 = \frac{\partial v}{\partial p_1} dp_1 + \frac{\partial v}{\partial I} dI \tag{6}$$

Equation (6) can be expressed as:

$$\frac{dI}{dp_1} = -\frac{\frac{\partial v}{\partial p1}}{\frac{\partial v}{\partial I}} \tag{7}$$

And finally substitute (4) and (5) into (7) to get the differential equation:

$$\frac{dI}{dp_1} = \frac{I}{2p_1} + \frac{1}{2} - \frac{p_2}{2p_1} \tag{8}$$

Rearrange (8) as:

$$\frac{dI}{dp_1} - \frac{I}{2p_1} = \frac{1}{2} - \frac{p_2}{2p_1} \tag{9}$$

Multiply (9) throughout by ($e^{-\frac{1}{2}\ln p_1}$) and integrate to get (10):

$$Ie^{-\frac{1}{2}\ln p_1} = p_1 e^{-\frac{1}{2}\ln p_1} + p_2 e^{-\frac{1}{2}\ln p_1} + C \tag{10}$$

Where C is the constant of integration, and its magnitude can easily be determined by substituting the original values of the other variables, namely; I=2000, p_1=20, p_2=10. And finally, (10) can be solved for I as a function in p_1, p_2 and C:

$$I = p_1 + p_2 + Ce^{\frac{1}{2}\ln p_1} \tag{11}$$

Equation (11) is the solution of the differential equation (9) and it expresses the relationship between I, p_1 and p_2, holding utility constant. Now effect a price change q_1=25. The compensating variation (CV) for the price change can be measured by the difference in the expenditure function (2):(1)

$CV = E(q,u) - E(p,u)$ =2237.53-2000=237.5

Using (11), however, the corresponding measure is:

$$CV = I(q, C) - I(p, C) = 2237.53 - 2000 = 237.5$$

Clearly, the approach by Hausman yields a similar value.

5.3. Equation (11) Versus (2)

Notwithstanding the similarity between (2) and (11), they are not the same. Hence, we cannot presume that (11) is the expenditure function which is the inverse of (1). It is clear that (11) operates on ordinary (Marshallian) demand function; It measures the ordinary demand (x) when income (I) and prices (p_1 and p_2) change, but holding utility constant. Consequently, (11) is a convenient substitute to (2) and serves well to measure the consumer surplus for the price change. For convenience call it the substitution function. However, a more general and exact method to calculate the change in expenditure holding utility constant is provided next.

5.4. A More General Analysis

In this section the analysis is generalized by considering two forms of utility functions: the Stone-Geary (LES) and the Cobb-Douglass.

a. The LES Analysis

It can be assumed that there are many commodities and the effect of a change in the prices on welfare is analyzed. However, to keep things simple without loss of generality assume one consumer only and two commodities. Hence it follows from Roy's identity and holding the consumer's utility constant that the change in income *(dI)* is equal to:

$$dI = x_1 dp_1 + x_2 dp_2 \tag{12}$$

Substitution for x_1 and x_2 in (12) yields:

$$dI = \frac{I + p_1 - p_2}{2p_1} dp_1 + \frac{I + p_2 - p_1}{2p_2} dp_2 \tag{13}$$

Rearranging (13) yields (14)

$$dI - \frac{I}{2p_1} dp_1 - \frac{I}{2p_2} dp_2 = \frac{p_1 - p_2}{2p_1} dp_1 + \frac{p_2 - p_1}{2p_2} dp_2 \tag{14}$$

(1) Microsoft excel sheet was used to obtain all the calculations in this paper. A simple calculater yields different magnitudes.

Therefore the solution of the differential equation (14) is given by (15):

$$I = Ce^{\frac{1}{2}\sum \ln p} + p_1 + p_2 \tag{15}$$

Where *C* denotes the constant of integration and can be evaluated by substituting in the corresponding values for *I, p_1* and *p_2*. Equation (15) is called the substitution function (for convenience) and it is a substitute for the expenditure function (2) which was obtained by finding the inverse of the indirect utility function (1). The latter being the solution for the maximization problem of the utility function. In equilibrium, the two functions are equal because the Hicksian and the Marshallian demands are identical.

To verify that (15) yields identical result as (2), effect a change in prices q_1=25 and q_2=15. And substitution for I=2000, p_1=20, p_2=10, a=b=1, α=β=0.5, in (15) yields a value for *C* equal to (139.3). Hence, (15) can be written as (16):

$$I = 139.3e^{\frac{1}{2}(\ln p_1 + \ln p_2)} + p_1 + p_2 \tag{16}$$

And substitution for the new prices yields a value for *I* equal to (2737.5). The corresponding value obtained by (2) is exactly the same. Also, to verify that (16) yields a result equal to that of (11) substitute the values p_1=25, p_2=10, to yield a value for I equal to (2237.53) which is again equal to the corresponding value given by (2).

On the other hand, the constant of integration *(C)* may be considered to correspond to a utility cardinal index. However, it is not related to neither the utility nor to the indirect utility functions, as has been misconceptually visualized by Hausman. It is evaluated as the paper demonstrated by substituting the original parameters.

b. The Cobb-Douglass Analysis

The corresponding indirect utility (v), expenditure (E) and Marshallian demand(x) functions for a Cobb-Douglass utility function of the form ($u=x_1^{\alpha}x_2^{\beta}$) are respectively:

$$v = \frac{I}{2(p_1p_2)^{\frac{1}{2}}}; E = 2v\,(p_1p_2)^{\frac{1}{2}}; x_i = \frac{I}{2p_i}$$

Assuming *I*=2000, *p_1*=20, *p_2*=10, the corresponding values are as follows: *v*=70.7, *x_1*=50, and *x_2*=100, respectively.

Similarly, a change in prices holding utility constant and using Roy's identity we have:

$$dI = \left\{ \frac{I}{2p_1} + \frac{I}{2p_2} \right\} dp_1$$

And its solution is given by:

$$I = Ce^{\frac{1}{2}\sum \ln p} \tag{17}$$

Substitution of the original values in (17) yields the magnitude (141.42) for *C*. Now affecting a change in the price of *x_1* to (25) yields the following answers for *(I)* and *(E),* respectively:

$$I = 141.42e^{\frac{1}{2}(\ln 25 + \ln 10)} = 2236.1$$

$$E = 2(70.7)(25*10)^{\frac{1}{2}} = 2236.1$$

It is clear from the above that both methods yield correct measurements. However, it is to be iterated that the solution of the differential equation *($dI=\sum xdp$)* is not the expenditure function. Albeit the similarity, the latter *(E(p,u))* is the inverse of the indirect utility function while the former *(S(p,I))* is derived from Marshallian demands. In other words the *(S(p,I))* does give precise values of *(V(P,I))*. The term *C*, the constant of integration, may serve as a utility cardinal index, but the inverse of *(S(p,I))* call it *(C(p,I))* is not the indirect utility function.

Notwithstanding the similarities between *(S(p,I))* and *(E(p,u))* they have different features and uses. The former may be identified as a substitution function while the latter is known as the expenditure function. The substitution function determines the rate of substitution between the control variables. And in this context, the expenditure function may be identified as a special case where the control variable is income. Solving the substitution function for *(I)* as a control variable determines how much compensation in monetary terms is needed to compensate the consumer for the price change. Perhaps viewing the problem from this angle caused some confusion and led to the misperception that the substitution function is the same as the expenditure function, as is clear from the paper by Hausman.

6. APPLICATION

In this section, actual data on disposable income and the price index are used to measure the effect of a 20% sales tax on welfare in Jordan. In 2004, disposable income (I) and the price index amounted to JD million

(2) Central Bank of Jordan, Monthly Statistical Bulletin, Oct. 2006.

(9691.1) and (113.6) respectively.[(2)] And substitution of these values in equation (11) yields the magnitude of (898.6) for C.

$$C = \frac{I - p}{e^{\frac{1}{2}\ln p}} = \frac{9691.1 - 113.6}{e^{\frac{1}{2}\ln 113.6}} = 898.6$$

And substitution of the after tax price (q=p(1.2)=136.32) in equation (11) yields the magnitude of 10627.9 for *I'* (the after tax value of *(I)* in equation (11) holding utility constant).

$$I' = 136.32 + 898.6 e^{-\frac{1}{2}\ln 136.32} = 10627.9$$

Hence the effect on welfare from the tax is given by:

$$dI = I' - I == 936.8$$

The effect on welfare is JD million (936.8).

7. CONCLUSION

This paper demonstrated that the welfare effect of a sales tax can be measured with accuracy, given the specified ordinary demand function (Marshallian), and it is not necessary to use the compensated demand (Hicksian). Using a specific demand function that is compatible with a LES utility function, the welfare loss of a 20% sales tax in Jordan amounted to JD million (936.8) in 2004.

Furthermore, this paper opened venues for further research in relevant areas to be undertaken. Different demand specifications, other than those used in this paper, can be employed. Differential taxes can be imposed and the resulting welfare loss can be investigated. Research can be undertaken to extract price and income elasticities of individual demands which can be used to specify demand and expenditure functions in a different manner and use them to measure welfare changes. Also, different population groups (low, middle, high income groups) can be introduced in the analysis instead of assuming one consumer.

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الخسارة الاجتماعية الناتجة عن فرض ضريبة المبيعات في الأردن

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ملخـص

تبين هذه الورقة كيفية قياس الخسارة الاجتماعية الناتجة عن فرض ضريبة مبيعات بنسبة 20% في الأردن عام 2004. وتحاول ان تسد الفجوة بين النظرية الاقتصادية الجزئية والدراسات التطبيقية في مجال اقتصاد الرفاه، وكما يبدو فإن الدراسات التطبيقية في هذا المجال عن الاردن قليلة جدا. بداية، تعتمد هذه الورقة على التحليل الذي اجراه Hausman (1981)) حيث اشتق دالة الانفاق من دالة الطلب العادية واستخدمها لقياس فائض المستهلك. فالورقة توضح التحليل الذي قام به Hausman، وتحاول إزالة بعض الغموض فيها باستخدام دالة منفعة محددة هي دالة ((Stone-Geary Utility Function) التي يطلق عليها مرادفة ((Linear Expenditure System (LES)) وشكلها العام ($u=(x_1+a)^{\alpha}\ (x_2+b)^{\beta}$)، ودالة الطلب العادية المرادفة لها. وانطلاقا من دالة الطلب تقوم الورقة باشتقاق معادلة تفاضلية، تسمى دالة الاحلال، التي تستخدم بدورها لقياس الخسارة الاجتماعية الناتجة عن فرض ضريبة مبيعات بنسبة 20% عام 2004 في الأردن.

تقوم الورقة باستعراض المفاهيم النظرية حول هذا الموضوع باختصار، وتوضح طريقة القياس باستخدام أمثلة معتمدة على دالتي المنفعة المشهورتين (Stone-Geary and Cobb-Douglass)، ومن ثم تعمم التحليل وتطبقه على الأردن. وتبين الورقة ان الخسارة الاجتماعية المترتبة على فرض ضريبة مبيعات بنسبة 20% في الأردن عام 2004 تصل الى 936 مليون دينار اعتمادا على طريقة قياس تستخدم دالة الطلب العادية. وتشير الورقة إلى أن المجال مفتوح لإجراء المزيد من الدراسات التطبيقية حول هذا الموضوع من خلال اعتماد اشكال أخرى من معادلات الطلب، وفرض ضرائب سلعية متباينة، واستخدام المرونات السلعية والدخلية، وتأثير الضرائب على مجموعات سكانية مختلفة.

الكلمـات الدالـة: قياس الخسارة الاجتماعية، ضريبة المبيعات، دالة الانفاق، فائض المستهلك، دالة الاحلال.

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